DESTINATION
MATH

Destination Math® Scope & Sequence

HOUGHTON MIFFLIN HARCOURT
SPECIALIZED CURRICULUM

TABLE OF CONTENTS

Destination Math® – Mastering Skills & Concepts I: Pre-Primary Mathematics, *Grades K-1* **3**

Destination Math – Mastering Skills & Concepts II: Primary Mathematics, *Grades 2-3* **5**

Destination Math – Mastering Skills & Concepts III: Basic Arithmetic, *Grades 4-6* **7**

Destination Math – Mastering Skills & Concepts IV: Intermediate Mathematics , *Grades 6-8* **10**

Destination Math – Mastering Skills & Concepts V: Pre-Algebra, *Grades 6-8* **13**

Destination Math – Mastering Algebra I: Course A, *Grades 8-10* **17**

Destination Math – Mastering Algebra I: Course B, *Grades 8-10* **19**

Destination Math Overview and Alignment with NCTM Standards **22**

Mastering Skills & Concepts I: Pre-Primary Mathematics

Skill Levels: Grades K–1
68 Learning Objectives

Designed for non-readers, this course lays a solid foundation as students learn important math concepts and skills they'll need to understand the world around them. In 30 sessions, they count from 1 to 100, identify shapes, add and subtract, compare and order numbers, and explore the concepts of time and money.

Module 1: Number Sense

Unit 1—Numbers from 1 to 5

Session 1 - Counting from 1 to 5
- Determining the number of objects in a set, from 1 to 3, and recognizing the corresponding numerals
- Determining the number of objects in a set, from 4 to 5, and recognizing the corresponding numerals

Session 2 - Creating Sets of 1 to 5
- Creating sets containing 1 to 3 objects
- Creating sets containing 4 to 5 objects

Session 3 - Creating Representations of the Numbers from 1 to 5
- Creating and recognizing representations of the numbers from 1 to 5

Unit 2 - Numbers from 1 to 10

Session 1 - Counting from 5 to 10
- Determining the number of objects in a set, from 6 to 8, and recognizing the corresponding numerals
- Determining the number of objects in a set, from 9 to 10, and recognizing the corresponding numerals

Session 2 - Creating Sets of 5 to 10
- Creating sets containing 5 to 10 objects

Session 3 - Creating Representations of the Numbers from 5 to 10
- Creating and recognizing representations of the numbers from 5 to 10

Session 4 - One More Than
- Naming the numbers that are one more than 1, 2, 3, 4, and 5
- Naming the numbers that are one more than 6, 7, 8, 9, and 10

Session 5 - One Fewer Than and Zero
- Naming the numbers that are one fewer than 5, 4, 3, 2, and 1
- Using '0' to represent the number of objects in an empty set
- Naming the numbers that are one fewer than 10, 9, 8, 7 and 6

Unit 3 - Numbers to 100

Session 1 - Counting from 10 to 20
- Recognizing the numerals and word names for the numbers from 11 to 15
- Using 10-frames to count and create sets of objects from 11 to 15
- Recognizing the numerals and word names for the numbers from 16 to 20
- Using 10-frames to count and create sets of objects from 16 to 20

Session 2 - Counting from 20 to 50
- Recognizing the numerals and word names for the numbers from 20 to 30
- Counting and creating sets of objects from 20 to 30
- Recognizing the numerals and word names for the numbers from 30 to 50
- Counting and creating sets of objects from 30 to 50

Session 3 - Counting from 50 to 100
- Recognizing the numerals and word names for the numbers from 50 to 100
- Using a hundreds chart to show the numbers from 50 to 100

Session 4 - Skip-Counting by Tens and Fives
- Skip-counting by tens and fives from 0 to 100

Session 5 - Skip-Counting by Twos
- Skip-counting by twos from 2 to 30
- Recognizing the even numbers up to 30
- Skip-counting by twos from 1 to 29
- Recognizing the odd numbers less than 30

Unit 4 - Comparing and Ordering

Session 1 - More Than, Less Than, or the Same
- Using one-to-one correspondence to compare equal and unequal sets

Session 2 - Comparing Numbers within 100
- Determining the inequality relationships between numbers from 1 to 10
- Using symbols to express the inequality relationships between numbers from 0 through 10
- Determining the inequality relationships between numbers from 10 to 100
- Using symbols to express the inequality relationships between numbers from 10 through 100

Module 2 - Addition and Subtraction

Unit 1 - Addition

Session 1 - Combining and Joining within 10
- Building number sentences to represent and solve combining and joining problems

Session 2 - Comparing within 10
- Building and completing number sentences when the second addend is unknown
- Building and completing number sentences when the first addend is unknown

Session 3 - Sums within 20, with 10 as One Addend
- Estimating solutions to addition story problems
- Building and completing number sentences with 10 as an addend

Session 4 - Sums within 20
- Estimating to predict a sum
- Determining the sum of two numbers up to 20
- Recognizing and expressing the sum of two one-digit numbers as the sum of 10 and another number

Unit 2 - Subtraction

Session 1 - Differences within 10
- Counting backwards to find the difference between two numbers
- Recognizing and completing number sentences involving differences within 10
- Using subtraction to solve comparison problems within 10

Session 2 - Differences within 20
- Building and completing number sentences involving differences within 20
- Recognizing and solving comparison problems involving differences within 20

Module 3 - Geometry and Measurement

Unit 1 - Measurement

Session 1 - Length
- Arranging objects in order by height and by length
- Using non-standard units to measure and compare lengths
- Using inches and centimeters to measure and compare length

Session 2 - Weight
- Using non-standard units to compare weights
- Arranging objects in order, by weight

Session 3 - Clock and Calendar Time
- Recognizing and using ordinal numbers
- Investigating the days of the week
- Using analog and digital clocks to tell time to the nearest hour and half-hour

Session 4 - Money
- Identifying pennies, nickels, dimes and quarters and their values
- Determining the amount of money represented by a set of pennies, nickels, dimes and quarters
- Determining the number and types of coins needed to represent a given amount of money

Unit 2 - Geometry

Session 1 - Triangles and Rectangles
- Interpreting a street map
- Identifying triangles
- Identifying rectangles
- Recognizing squares as special rectangles

Session 2 - Three-Dimensional Shapes
- Exploring common three-dimensional shapes and their two-dimensional nets
- Identifying the faces of common three-dimensional shapes

Module 4 - Algebraic Thinking

Unit 1 - Patterns and Displays

Session 1 - Shapes
- Recognizing, completing, and extending linear patterns involving shapes
- Representing linear patterns using letters, such as ABC or BAC
- Using Venn diagrams to sort shapes according to one or more properties

Session 2 - Number Patterns
- Recognizing, completing, and extending number patterns.
- Identifying missing terms in an addition or subtraction sequence

Session 3 - Tables and Graphs
- Sorting and representing data in a picture graph
- Analyzing data in a picture graph
- Using tally marks to create a frequency table
- Representing and interpreting data in a bar graph

Mastering Skills & Concepts II: Primary Mathematics

Skill Levels: Grades 2-3
69 Learning Objectives

In this course, students practice place value, rounding, fractions, planes and solid figures, addition, subtraction, multiplication, and division. In addition, they hone their skills by measuring time, money, and temperature.

Module 1 - Number Sense

Unit 1 - Numbers to 999

Session 1 - Counting by Grouping
- Counting a set of objects by grouping them into tens and ones
- Recognizing and writing equivalent base-10 names for a number

Session 2 - Place Value: Tens and Ones
- Representing the standard form of a number in terms of tens and ones
- Identifying the standard form of a number given its place value representation

Session 3 - Place Value: Hundreds, Tens, and Ones
- Using base-10 blocks to represent a 3-digit number
- Identifying the value of each place in a 3-digit number
- Recognizing the word name of a 3-digit number

Session 4 - Expanded Form and Equivalent Representations of a Number
- Creating equivalent representations of a 2-digit number by regrouping ones and tens
- Expressing a 2-digit number in expanded form
- Creating equivalent representations of a 3-digit number by regrouping ones and tens
- Expressing a 3-digit number in expanded form

Session 5 - Comparing and Ordering
- Using inequality signs to compare 2-digit numbers
- Determining the order of two or three non-consecutive numbers less than 100
- Using inequality signs to compare two 3-digit numbers, and a 2-digit and a 3-digit number
- Determining the order of up to three non-consecutive numbers less than 1,000

Unit 2 - Numbers to 9,999

Session 1 - Place Value: Thousands, Hundreds, Tens, and Ones
- Using base-10 blocks to determine the value of each place in a 4-digit number
- Expressing a 4-digit number in expanded form
- Recognizing the word name of a 4-digit number

Session 2 - Comparing and Ordering
- Using inequality signs to compare 3-digit and 4-digit numbers
- Ordering numbers on a number line

Module 2 - Operations with Numbers

Unit 1 - Addition and Subtraction

Session 1 - Sums less than 100
- Finding the sum of a 2-digit number and a 1-digit number without regrouping
- Using regrouping to find the sum of two 2-digit numbers
- Recognizing that the order of two addends does not affect their sum

Session 2 - Estimating and Finding Sums Less Than 1,000
- Using a number line to estimate the sum of two 3-digit numbers
- Regrouping in the ones place to find the sum of two 3-digit numbers
- Regrouping in the tens and ones places to find the sum of two 3-digit numbers

Session 3 - Differences within 100
- Using regrouping to subtract a 1-digit number from a 2-digit number
- Using regrouping to subtract a 2-digit number from a 2-digit number

Session 4 - Estimating and Finding Differences within 1,000
- Using a number line to estimate the difference between a 3-digit number and a 2-digit number
- Regrouping in the hundreds place to estimate and find the difference between a 3-digit number and a 2-digit number
- Checking subtraction using addition

Session 5 - Estimating and Finding Differences within 9,999
- Estimating the difference between a 4-digit number and a 3-digit number
- Using base-10 blocks to represent the subtraction of a 3-digit number from a 4-digit number
- Applying the subtraction algorithm to find the difference between a 4-digit number and a 3-digit number

Unit 2 - Multiplication

Session 1 - Repeated Addition and Arrays
- Recognizing and writing multiplication sentences to represent repeated addition
- Recognizing and writing multiplication sentences to represent objects in a rectangular array
- Recognizing that the order of two factors does not affect their product

Session 2 - Skip Counting to Show Multiplication
- Solving comparison problems by skip counting by numbers less than 10
- Recognizing and using multiplication sentences to show multiples of a measure on a number line within 100
- Solving comparison problems by skip counting by 10
- Recognizing and using multiplication sentences to show multiples of a measure on a number line within 1,000

Session 3 - Finding Products less than 100
- Exploring a rectangular array to determine products up to 100
- Using base-10 blocks to find the product of a 2-digit number and a 1-digit number
- Applying the multiplication algorithm to find the product of a 2-digit number and a 1-digit number

Unit 3 - Division

Session 1 - Meaning of Division
- Finding a quotient using the concept of equal groups
- Finding a quotient using the concept of repeated subtraction
- Identifying the remainder in a division problem

Session 2 - Dividing by a 1-digit Number
- Using base-10 blocks to find the quotient of a 2-digit number and a 1-digit number
- Using base-10 blocks to find the quotient of a 3-digit number and a 1-digit number
- Checking a quotient using multiplication

Unit 3 - Division (cont'd)

Session 3 - Fractional Parts
- Identifying and naming equal parts of a whole
- Using a fraction to express part of a whole
- Using fractions to represent and compare parts of a group

Module 3 - Geometry and Measurement

Unit 1 - Geometry

Session 1 - Area
- Estimating the area of a shape using non-standard units
- Finding the area of a shape using standard units
- Comparing the areas of two or more shapes using standard units

Session 2 - Volume
- Using standard units to compare the capacity of two or more containers
- Using cubic units to compare the volume of two solids

Session 3 - Perimeter
- Use non-standard units to measure perimeter
- Use inches and centimeters to measure perimeter

Session 4 - Congruence and Symmetry
- Identifying congruent and non-congruent shapes
- Exploring symmetry and lines of symmetry

Unit 2 - Measurement

Session 1 - Time
- Telling time to the nearest minute before and after the hour
- Working with start time, end time, and elapsed time

Session 2 - Money
- Recognizing and using decimal notation to express the value of U.S. currency
- Determining and comparing values of combinations of bills and coins less than 10 dollars
- Using a counting up strategy to make change within 10 dollars

Session 3 - Temperature
- Showing temperatures on a Fahrenheit scale and on a Celsius scale
- Solving problems involving changes in temperature in degrees Fahrenheit or in degrees Celsius

Session 4 -Factional Units and Length
- Measure length using customary units to the nearest inch, half inch, and quarter inch
- Measure Length using metric units to the nearest centimeter and millimeter

Module 4 - Algebraic Thinking

Unit 1 - Properties and Relationships

Session 1 - Number Patterns and Properties
- Recognizing and applying the commutative properties of addition and multiplication
- Recognizing and applying the associative properties of addition and multiplication
- Choosing the correct number or sign to complete numeric equations
- Exploring a linear pattern between two quantities

Mastering Skills & Concepts III: Intermediate Mathematics

Skill Levels: Grades 4-6
125 Learning Objectives

This course focuses on the numbers operations typically included in a math curriculum for grades 4-6, but provides educators with enough flexibility to use it above or below the recommended grade range. Students investigate how mathematical issues arise out of real-life situations. Within motivational contexts, students work through tutorials designed around 125 learning objectives in Numbers and Number Sense, Operations with Numbers, Fractions, Decimals, Geometry, and Data Analysis and Probability.

Module 1 - Numbers and Number Sense
Unit 1 - Large and Small Numbers
Session 1 - Whole Numbers to One Million
- Using 10 to generate the pattern of numbers 1, 10, 100, 1,000, 10,000, 100,000, and 1,000,000 and to represent them in standard and word form
- Expanding the place value grid up to 1,000,000
- Representing a number up to one million in expanded form and as the product of each digit times its place value
- Writing the word names of numbers up to a million

Session 2 - Ordering and Rounding Whole Numbers
- Comparing and ordering large numbers using place value grids and/or number lines
- Using equality or inequality signs to express the relationship between two whole numbers
- Rounding whole numbers down to specified place values
- Rounding whole numbers up to specified place values

Session 3 - Negative Whole Numbers
- Graphing positive and negative whole numbers on a number line
- Comparing two or more integers using statements involving $<$, $>$, and $=$
- Rounding negative integers to specified place values

Unit 2 - Numbers as Factors
Session 1 - Finding Factors
- Using an area model to represent multiplication
- Demonstrating that multiplication is commutative
- Finding the pairs of factors of a whole number
- Recognizing that any number has 1 and itself as factors

Session 2 - Prime and Composite Numbers
- Identifying the prime numbers less than 50
- Determining the prime factors of a number

Session 3 - Identifying Common Factors
- Finding the common factors of two whole numbers
- Using factor trees and a Venn diagram to identify the greatest common factor of two 2-digit numbers
- Finding the greatest common factor of two 3-digit numbers

Module 2 - Operations with Numbers
Unit 1 - Addition and Subtraction of Whole Numbers
Session 1 - Whole Number Sums
- Estimating the sum of two or more 3-, 4-, and 5-digit numbers
- Finding the sum of two or more 3-, 4-, and 5-digit numbers
- Checking an addition by using the Commutative Property of Addition

Session 2 - Differences Between Large Numbers
- Using regrouping to subtract two 4-digit numbers
- Checking the difference by addition
- Using regrouping to subtract two 5-digit numbers
- Checking the difference by addition

Unit 2 - The Integers
Session 1 - Integer Sums
- Finding the sum of two positive whole numbers using a number line
- Finding the sum of two negative whole numbers
- Finding the sum of a positive and negative whole number

Session 2 - Differences Between Integers
- Recognizing that the sum of two opposites is 0
- Representing the sum of two integers using colored chips
- Finding the difference between a negative integer and a positive integer
- Checking a difference using addition

Unit 3 - Multiplication and Division of Whole Numbers
Session 1 - Two-digit Multipliers
- Modeling the product of a 2-digit number and a 1-digit number using the areas of rectangles
- Applying the Distributive Property to multiply two numbers
- Using the multiplication algorithm to find the product of two 2-digit numbers
- Checking a product using the Commutative Property of Multiplication

Session 2 - Introduction to Long Division
- Modeling the quotient of a 3-digit number and a 1-digit number using areas of rectangles
- Estimating a quotient by locating it between consecutive multiples of 10
- Checking the division by multiplying the quotient and the divisor
- Using the division algorithm to divide a 3-digit number by a 1-digit number without a remainder

Session 3 - Two-digit Divisors
- Divide a 4-digit number by a 2-digit number
- Identify the remainder in a division problem

Unit 4 - Simplifying Expressions
Session 4 - Exponents and Order of Operations
- Explore different representations of exponents
- Use the order of operations to simplify numerical expressions

Module 3 - Fractions
Unit 1 - Proper and Improper Fractions
Session 1 - Proper Fractions
- Plotting unit fractions on the number line
- Plotting proper and improper fractions on the number line

Unit 1 - Proper and Improper Fractions (cont'd)

Session 2 - Improper Fractions
- Investigating improper fractions
- Expressing an improper fraction as a mixed number
- Plotting improper fractions and mixed numbers on a number line

Session 3 - Equivalent Fractions
- Using a circle graph to represent fractions
- Reducing a fraction to lowest terms
- Using the Multiplication Property of One to rewrite a given fraction as an equivalent fraction

Session 4 - Ordering and Rounding Fractions
- Finding a common denominator for two fractions
- Comparing and ordering two fractions
- Rounding a fraction to the nearest whole number

Unit 2 - Addition and Subtraction

Session 1 - Sums Involving Like Denominators
- Estimating the sum of two fractions to the nearest whole number
- Calculating the sum of two fractions having like denominators
- Estimating the sum of fractions and mixed numbers to the nearest whole number
- Calculating the sum of fractions and mixed numbers

Session 2 - Differences Involving Like Denominators
- Calculating the difference between two simple fractions having like denominators
- Estimating the difference between two mixed numbers to the nearest whole number
- Calculating and checking the difference between two mixed numbers

Session 3 - Working with Unlike Denominators
- Identifying a common denominator for fractions that have unlike denominators
- Estimating and calculating the sum or difference of fractions having unlike denominators
- Estimating and calculating the sum or difference of mixed numbers whose fractional parts have unlike denominators

Unit 3 - Multiplication and Division

Session 1 - Finding Products
- Calculating the products of proper and improper fractions
- Calculating products of fractions and mixed numbers
- Estimating the products of two fractions

Session 2 - Quotients and Remainders
- Dividing a whole number by a proper fraction
- Estimating the quotient of two mixed numbers or improper fractions
- Dividing two mixed numbers or improper fractions

Module 4 - Decimals

Unit 1 - Introduction

Session 1 - Tenths, Hundredths, and Thousandths
- Using a place value grid to explore 1-place decimal numbers
- Representing tenths in standard form, expanded form, and word form
- Using a place value grid to explore 2- and 3-place decimal numbers
- Representing hundredths and thousandths in standard form, expanded form, and word form

Session 2 - Ordering and Rounding
- Rounding decimals to the nearest tenth
- Representing data on a bar graph
- Comparing and ordering two or more decimals

Session 3 - Ratios, Decimals, and Percents
- Expressing ratios as percents
- Expressing a decimal as a percent
- Expressing the equivalences among ratios, decimals, and percents

Unit 2- Addition and Subtraction

Session 1 - Adding Decimals
- Estimating the sum of two or more decimals by rounding the decimals to the nearest whole number
- Adding decimal numbers in tenths and hundredths without regrouping
- Adding decimal numbers in tenths, hundredths, and thousandths with regrouping
- Checking an addition of decimals using their fractional equivalents

Session 2 - Subtracting Decimals
- Estimating the difference between two decimal numbers by rounding each decimal to the nearest whole number
- Using regrouping to find the difference between two decimal numbers in tenths and hundredths, and checking using addition
- Using regrouping to find the difference between two decimal numbers in thousandths, and checking using addition

Unit 3 - Multiplication and Division

Session 1 - Multiplying Decimals
- Estimating and finding the product of a decimal and a whole number
- Estimating and finding the product of two decimals
- Inserting zeros in a product to place the decimal point
- Checking the product of two decimals using fractional equivalents

Session 2 - Dividing Decimals by Whole Numbers
- Estimating the answer of a decimal number greater than 1 divided by a whole number
- Dividing a decimal number greater than 1 by a whole number that is less than the dividend, and checking the answer by multiplication
- Estimating the answer of a decimal number less than 1 divided by a whole number
- Dividing a decimal number greater than 1 by a whole number that is greater than the dividend, and checking by multiplication

Module 5 - Geometry

Unit 1 - Measurement

Session 1 - Lines, Angles, and Circles
- Exploring lines, segments, rays, and angles
- Classifying angles
- Using a protractor

Session 2 - Rectangles and Squares
- Examining the properties of a rectangle and a square
- Defining perpendicular and parallel lines
- Calculating the perimeters of rectangles and squares
- Exploring the relationship between the perimeters and areas of rectangles and squares

Session 3 - Triangles
- Classifying triangles according to the measure of their sides
- Determining that the sum of the angles of a triangle equals 180 degrees
- Finding the perimeter of a triangle
- Classifying triangles according to the measures of their angles

Session 4 - Parallelograms and Trapezoids
- Exploring the properties of a parallelogram
- Discovering the area formula of a parallelogram
- Exploring the properties of a trapezoid
- Discovering the area formula of a triangle

Session 5 - Converting Units
- Use multiplication to convert customary units
- Use division to convert customary units
- Use multiplication and division to convert metric units

Session 6 - Measurement Tools, Units, and Precision
- Choose and appropriate unit and measurement tool
- Explore measurement and precision

Unit 2 - Coordinate Geometry and Algebra

Session 1 - The Coordinate Plane
- Plotting and reading ordered pairs in a coordinate plane
- Finding vertical and horizontal distances between points in a coordinate plane
- Finding perimeters and areas of polygons graphed in a coordinate plane

Session 2 - Symmetry and Transformations
- Exploring line symmetry and reflections in the coordinate plane
- Exploring translations in the coordinate plane
- Exploring rotations in the coordinate plane

Session 3 - Algebraic Expressions and Functions
- Use an input output table to represent a function
- Write an algebraic expression for a function

Unit 3 - Two-and-Three Dimensional Shapes

Session 1 - Relating Two-and Three Dimensional Shapes
- Explore and classify solid shapes
- Relate solid shapes and their nets

Module 6 - Data Analysis and Probability

Unit 1 - Modeling and Displaying Events

Session 1 - Displaying and Analyzing Data
- Creating and analyzing the graphs of sets of one-dimensional data
- Finding the mean, median, and mode in a set of one-dimensional data
- Graphing and analyzing sets of two-dimensional data

Session 2 - Looking at Chance
- Using a tree diagram to represent the outcomes in a probability experiment
- Representing the frequencies, ratios, and percentages of outcomes in a simple probability experiment
- Determining the probability of two independent outcomes

Mastering Skills & Concepts IV: Basic Mathematics

Skill Levels: Grades 6-8
148 Learning Objectives

Focusing on the numbers and operations typically included in the middle school math curriculum, this course presents each topic within a motivational context that demonstrates how mathematical issues arise out of real-life situations. Within these contexts, students investigate the properties of fractions, decimals, percents, and integers, and explore the rules that govern their operations.

Module 1 - Fractions

Unit 1 - Essentials of Fractions

Session 1 - Recognizing a Fraction
- Learning that a fraction is a part of a whole through the use of area models
- Identifying the numerator and denominator of a fraction
- Identifying fractional parts of a whole number based on a diagram

Session 2 - Exploring Proper and Improper Fractions
- Comparing the magnitudes of two or more fractions
- Adding fractions with like denominators
- Expressing 1 as equivalent fractions with equal numerators and denominators
- Identifying proper and improper fractions
- Writing an integer as a fraction with a denominator of 1
- Recognizing a fraction as the division of two numbers
- Using division to express improper fractions as mixed numbers

Session 3 - Working with Mixed Numbers
- Identifying mixed numbers
- Writing a mixed number as an improper fraction
- Identifying different types of fractions
- Comparing proper fractions and improper fractions

Unit 2 – Equivalent Fractions

Session 1 - Identifying the Factors of a Number
- Identifying a proper fraction and the parts of a fraction
- Modeling a proper fraction using a circle
- Finding the factors of a fraction's numerator and denominator
- Identifying the common factors of a fraction's numerator and denominator

Session 2 - Expressing Fractions in Lowest Terms
- Expressing a fraction in lowest terms
- Naming equivalent fractions
- Identifying equivalent fractions using circle graphs

Session 3 - Writing and Comparing Equivalent Fractions
- Writing two or more fractions with different denominators in terms of the same denominator
- Comparing fractions with different denominators

Unit 3 - Multiplying Fractions

Session 1 - Finding Products of Fractions, Whole Numbers, and Mixed Numbers
- Writing fractions in lowest terms
- Multiplying proper fractions and whole numbers
- Multiplying proper fractions and mixed numbers
- Multiplying fractions by multiplying numerators together and denominators together
- Using a number line to compare fractions

Session 2 - Using the GCF in Finding Products
- Multiplying proper fractions and mixed numbers
- Using the GCF to cancel like factors in a product
- Multiplying improper fractions and whole numbers
- Solving multi-step problems involving multiplication of fractions and whole numbers

Session 3 - Representing Multiplication
- Representing the products of proper fractions and mixed numbers using area models
- Rewriting a multiplication problem in terms of addition
- Representing the products of proper fractions and mixed numbers using area models
- Rewriting the product of two mixed numbers using the distributive property of multiplication
- Checking solutions to multiplication problems solved using the distributive property of multiplication

Unit 4 - Dividing Fractions

Session 1 - Estimating Quotients of Fractions
- Setting up a division problem involving mixed numbers and proper fractions
- Identifying the divisor, dividend, and quotient in a division problem
- Using a number line to round a fraction to the nearest whole number
- Estimating the quotient of two fractions by rounding each fraction to the nearest whole number

Session 2 - Using Multiplicative Inverses
- Finding the multiplicative inverse, or reciprocal, of a number
- Dividing two fractions by multiplying the dividend by the multiplicative inverse of the divisor
- Interpreting an answer so that it satisfies a condition of the problem

Session 3 - Solving Missing Value Problems when Dividing Fractions
- Writing whole numbers as fractions with denominators of 1
- Dividing whole numbers, mixed numbers, and proper fractions
- Using trial and error to find an unknown divisor when the dividend and quotient are given
- Using trial and error to find an unknown dividend when the divisor and quotient are given

Unit 5 - Adding Fractions

Session 1 - Adding with Like Denominators
- Adding mixed numbers and proper fractions with like denominators
- Using number lines to show addition of mixed numbers and proper fractions
- Rounding two or more mixed numbers to estimate their sum

Session 2 - Adding with Unlike Denominators
- Finding the least common multiple of two or more numbers
- Adding mixed numbers by adding the whole number parts and fractional parts
- Identifying the least common denominator of two or more fractions
- Rewriting fractions with unlike denominators as fractions with like denominators

Session 3 - Solving Missing Value Problems when Adding Fractions
- Adding mixed numbers with unlike denominators
- Finding the missing addend in an addition problem
- Finding the missing addend using the inverse operation of subtraction

Unit 6 - Subtracting Fractions

Session 1 - Subtracting with Like Denominators
- Subtracting mixed numbers with like denominators
- Expressing the area of a polygonal region as the sum of the areas of two rectangles
- Expressing the lengths of the sides of a rectangle as differences between given lengths

Session 2 - Subtracting with Unlike Denominators
- Subtracting mixed numbers with unlike denominators
- Calculating the area of a polygonal region

Session 3 - Solving Missing Value Problems when Subtracting Fractions
- Subtracting mixed numbers with unlike denominators
- Checking solutions to subtraction problems
- Representing subtraction using line segments
- Finding missing values in subtraction problems

Module 2 - Decimals

Unit 1 - Essentials of Decimals

Session 1 - Investigating Decimal Place Values
- Comparing the decimal system to some other number systems
- Identifying place values in the decimal system
- Expressing mixed numbers as decimals
- Labeling places to the right of the decimal point as tenths, hundredths, thousandths, and ten-thousandths
- Using zero as a place holder

Session 2 - Rounding Decimals
- Defining the meaning of Pi in terms of a diameter and circumference of a circle
- Rounding decimal numbers to two, three, and four decimal places
- Locating rounded decimal values on a number line

Session 3 - Exploring Repeating and Terminating Decimals
- Using division to write fractions whose denominators are not factors of 10 as equivalent decimals
- Identifying repeating, non-terminating decimal numbers and non-repeating, non-terminating decimal numbers
- Recognizing and using symbols to represent repeating and non-repeating, non-terminating decimal numbers
- Rounding repeating, non-terminating decimal numbers
- Ordering numbers that have different numbers of decimal places

Unit 2 - Adding and Subtracting Decimals

Session 1 - Using Place Value Grids
- Rounding dollars and cents to the nearest whole dollar amount
- Using a hundredths place value grid to add decimal numbers
- Arranging terms when adding according to their decimal points and corresponding place value positions

Session 2 - Regrouping with Whole Numbers
- Subtracting decimals in terms of dollars and cents
- Arranging terms when subtracting according to their decimal points and corresponding place value positions
- Regrouping whole numbers to subtract decimals

Session 3 - Regrouping to Hundredths
- Arranging terms when subtracting according to their decimal points and corresponding place value positions
- Subtracting decimal numbers by regrouping whole numbers, tenths, and/or hundredths

Unit 3 - Multiplying Decimals

Session 1 - Multiplying Decimals by Powers of 10
- Using fractions to multiply two-place decimals by 10, 100, and 1,000
- Multiplying decimals by rewriting each factor as a fraction and multiplying the fractions
- Using a shortcut to move the decimal point when multiplying by a power of 10

Session 2 - Calculating Products
- Using a shortcut to move the decimal point when multiplying by a power of 10
- Using scale models to calculate dimensions of a rectangular prism
- Multiplying decimal factors
- Recognizing the formula $V = lwh$ for finding the volume of a rectangular prism

Session 3 - Finding the Volume of a Prism
- Calculating the volume of a rectangular prism
- Multiplying decimals by rewriting each factor as a fraction and multiplying the fractions
- Using the correct units when calculating volume

Unit 4 - Dividing Decimals

Session 1 - Dividing Decimals by Whole Numbers
- Dividing a decimal number by a whole number
- Checking the quotient of a division calculation

Session 2 - Estimating and Finding Quotients
- Expressing a decimal denominator as a whole number by multiplying the numerator and denominator of the fraction by a power of 10
- Dividing a decimal number by a decimal number
- Adding zeros to the right of a decimal point to act as place holders in a dividend
- Estimating an answer when dividing by decimals

Session 3 - Dividing by Powers of 10
- Using the prefix 'kilo' to express metric units of thousands
- Dividing a decimal number by a power of 10
- Expressing a decimal denominator as a whole number by multiplying the numerator and denominator of the fraction by a power of 10
- Finding the quotient of a proper fraction

Module 3 - Percents

Unit 1 - Essentials of Percents

Session 1 - Investigating the Meaning of Percent
- Comparing the magnitude of 2 fractions whose LCD is 100
- Rewriting proper fractions with denominators of 100 as percents
- Using division to rewrite proper fractions as percents

Session 2 - Expressing Percents as Proper Fractions
- Converting percents less than 100% to fractions
- Ordering percents by magnitude

Session 3 - Expressing Percents greater than 100% as Improper Fractions
- Interpreting and using a pie chart (circle graph) to represent percents
- Expressing percents greater than 100% as improper fractions

Unit 2 - Finding Percents of Quantities

Session 1 - Finding Percents of a Whole
- Finding the part given the percent and the whole
- Finding the percent represented by the ratio of a part and a whole

Session 2 - Expressing Ratios as Percents
- Finding the percent represented by the ratio of a part and a whole
- Expressing a decimal as a percent

Session 3 - Calculating the Whole from a Part and a Percent
- Finding the whole given a percent and a part
- Expressing a percent as a decimal

Unit 3 - Increasing and Decreasing Percents

Session 1 - Calculating Percent Increases
- Increasing a number by multiplying it by a percent and adding the increase to the original amount
- Increasing a number by multiplying it by 100% plus the percent increase

Session 2 - Calculating Percent Decreases
- Decreasing a number by multiplying it by 100% less the percent decrease
- Decreasing a number by multiplying it by a percent and subtracting the decrease from the original amount

Session 3 - Calculating Simple Interest
- Calculating simple interest on a loan using the formula: Interest = Principal x Rate x Time
- Comparing the interest added to a given principal loaned over 2 separate periods of time

Module 4 - Integers and Order of Operations

Unit 1 - Adding and Subtracting Signed Numbers

Session 1 - Exploring the Number Line and Absolute Value
- Plotting negative numbers on a number line
- Using a number line to represent the meaning of the absolute value of numbers
- Recognizing integers, whole numbers, and counting numbers

Session 2 - Adding with Absolute Value
- Finding the sum of two or more signed numbers
- Using a number line to add two integers
- Using absolute value to find the sum of two integers

Session 3 - Subtracting with Absolute Value
- Using a number line to subtract two integers
- Recognizing subtraction as the addition of opposites

Unit 2 - Multiplying and Dividing Signed Numbers

Session 1 - Finding Products of Signed Numbers
- Multiplying integers having unlike signs
- Multiplying negative integers
- Using a pattern to discover the rules for multiplying negative integers

Session 2 - Representing the Multiplication of Signed Numbers
- Dividing negative decimals by multiplying the dividend by the inverse of the divisor
- Writing the multiplicative inverse of a decimal number

Session 3 - Finding Quotients Using Reciprocals
- Dividing negative decimals
- Dividing integers having unlike signs

Unit 3 - Order of Operations

Session 1 - Simplifying Expressions
- Recognizing the order of operations: PEMDAS or Please Excuse My Dear Aunt Sally
- Recognizing the meaning of integer exponents

Session 2 - Introducing the Distributive Property
- Writing expressions that represent a given situation
- Applying the distributive property of multiplication over addition

Session 3 - Using Grouping Symbols
- Recognizing parentheses, brackets, and fraction bars as grouping symbols
- Using more than one pair of grouping symbols in an expression

Mastering Skills & Concepts: Course V: Pre-Algebra

Skill Levels: Grades 6-8
194 Learning Objectives

This course prepares students for the more formal study of mathematics in high school. Students continue their course of study of numbers and operations by exploring ratios, proportions, and irrational numbers. They also begin a study of fundamental skills and concepts found in algebra, geometry, statistics, and probability. Students apply what they have learned to solve sets of questions at varying levels of difficulty.

Module 1 - Essentials of Algebra

Unit 1 - Algebra Fundamentals

Session 1 - Introducing Variables
- Rewriting the formula for the volume of a rectangular prism by substituting expressions for each term
- Using variables to represent the terms in the formula for the volume of a rectangular prism

Session 2 - Identifying Components of Algebraic Expressions
- Identifying the coefficient in a variable expression
- Identifying the constant in an expression
- Identifying an algebraic term
- Identifying an algebraic expression

Session 3 - Replacing Variables in a Formula
- Substituting known values for the variables in an expression
- Calculating the volume of a rectangular prism given the value of its dimensions

Unit 2 - Evaluating an Algebraic Expression

Session 1 - Representing the Dimensions & Area of a Rectangle
- Representing the dimensions of a rectangle in terms of l and w
- Representing the areas of rectangles using variable expressions

Session 2 - Combining Like Terms
- Applying the commutative property of multiplication
- Applying the distributive property of multiplication over addition
- Simplifying expressions by combining like terms
- Simplifying expressions by using the order of operations

Session 3 - Evaluating Expressions Using Substitution
- Subtracting polynomial expressions
- Substituting values of known quantities for variables in expressions

Unit 3 - Simple Equations

Session 1 - Using Variables to Express Relationships
- Choosing variables to represent each of the unknown quantities in a problem
- Using algebraic expressions to show the relationship between variables
- Substituting one variable for another and writing an equation containing only one variable term

Session 2 - Simplifying Algebraic Expressions
- Simplifying one side of an equation using the distributive property of multiplication over addition and following the order of operations
- Combining like terms
- Investigating the elements of an algebraic expression

Session 3 - Solving Simple Equations
- Balancing an equation
- Isolating a variable by adding or subtracting a constant from both sides of the equation
- Multiplying or dividing both sides of an equation by the coefficient of the variable to solve the equation
- Checking a solution by substituting the value of a variable into the equation used to solve it
- Solve a two-step equation using inverse operations

Unit 4 - Variable on Both Sides of the Equation

Session 1 - Writing Equations
- Using a variable to represent an unknown quantity in a problem
- Using the same variable to represent a 2nd unknown quantity
- Writing an equation that represents the conditions of the problem
- Simplifying each side of an equation

Session 2 - Simplifying Both Sides of an Equation
- Collecting the variable terms on one side of the equation
- Isolating the variable term

Session 3 - Checking the Solution to an Equation
- Solving for the variable
- Checking the solution in the original equation
- Checking that the solution is complete and satisfies the conditions in the problem

Unit 5 - Solving Literal Equations

Session 1 - Identifying the Variables in a Given Formula
- Identifying the variables in the formula for the volume of a frustum of a cone
- Recognizing the radius and diameter of a circle
- Using substitution to express one radius in terms of the other
- Simplifying algebraic expressions by multiplying and combining like terms

Session 2 - Rewriting a Formula in Terms of a Different Variable
- Using the properties of equality to rewrite a formula for a particular variable

Session 3 - Substituting Values & Solving an Equation
- Substituting values in a literal equation to solve for a particular variable
- Applying the order of operations to simplify expressions
- Checking a solution in the original formula

Module 2 - Fundamentals of Geometry

Unit 1 - Geometry Fundamentals

Session 1 - Naming and Measuring Angles

- Defining a right angle
- Using a protractor to measure angles
- Knowing the meaning of 'perpendicular
- Recognizing a parallelogram as a four-sided figure having opposite parallel sides
- Using a protractor to measure angles
- Recognizing a straight angle
- Naming angles
- Defining obtuse angles

Session 2 - Defining Complementary & Supplementary Angles

- Defining an acute angle
- Defining supplementary angles
- Defining complementary angles
- Writing equations to show relationships between angles

Session 3 - Recognizing Congruent Angles

- Recognizing supplementary angles
- Defining congruent angles
- Defining vertical angles
- Establishing congruence between pairs of angles
- Identifying pairs of alternate interior and alternate exterior angles

Unit 2 - Triangles

Session 1 - Classifying Triangles by Sides

- Dissecting a quadrilateral into sets of triangles
- Defining a right triangle
- Defining an isosceles triangle
- Defining a scalene triangle

Session 2 - Exploring the Area of a Triangle

- Relating the area of a triangle to the area of a rectangle
- Identifying the height of a triangle
- Calculating the area of a triangle
- Defining an equilateral triangle

Session 3 - Classifying Triangles by Angles

- Applying the triangle sum formula to find missing angle measures
- Identifying right triangles
- Identifying acute triangles
- Identifying obtuse triangles

Unit 3 - Volume and Surface Area

Session 1 - Calculating the Volume of a Right Triangular Prism

- Classifying a prism according to its base
- Identifying right prisms
- Expressing the volume of a right triangular prism: $V = (1/2)\ b \times h$
- Calculating the volume of a right triangular prism

Session 2 - Calculating the Surface Area of a Right Triangular Prism

- Defining the surface area of an object
- Defining the faces of a right triangular prism
- Recognizing a foldout for a right triangular prism
- Calculating part of the surface area of a right triangular prism

Session 3 - Calculating the Volume & Surface Area of a Right Cylinder

- Calculating the volume of a right cylinder
- Calculating the circumference of a circle
- Calculating the surface area of a right cylinder

Module 3 - Radicals & Exponents

Unit 1 - Introduction to Radicals & Pythagorean Theorem

Session 1 - Exploring the Pythagorean Theorem

- Identifying the hypotenuse in a right triangle
- Using variables to represent the Pythagorean Theorem
- Identifying a right triangle given the measure of its sides

Session 2 - Investigating Squares & Square Roots

- Completing a table of square numbers up to 12
- Determining the square roots of some perfect squares
- Plotting squares and square roots on a number line
- Investigating cubing a number and cube roots with reference to the volume of a cube

Session 3 - Defining Irrational Numbers

- Finding the length of the third side of a right triangle given the measures of two sides
- Locating the square root of a number between consecutive integers
- Recognizing irrational numbers as non-terminating, non-repeating decimals
- Classifying numbers as either rational or irrational

Unit 2 - Introduction to Scientific Notation

Session 1 - Writing Numbers Using Scientific Notation

- Writing a number using scientific notation

Session 2 - Comparing Numbers in Scientific Notation

- Converting numbers from standard form to scientific notation
- Recognizing that 1 kilo is equal to 10^2
- Using the on-line calculator to express numbers in scientific notation
- Comparing two numbers written in scientific notation

Session 3 - Writing Numbers between zero & one in Scientific Notation

- Writing a number between 0 and 1 in scientific notation
- Exploring powers of 10 that are negative integers and 0
- Converting numbers from scientific notation to standard form

Module 4 - Ratio & Proportion

Unit 1 - Ratio

Session 1 - Defining Ratio
- Defining the terms and symbols of a ratio
- Expressing a ratio in lowest terms
- Recognizing equivalent ratios

Session 2 - Expressing Ratios as Equivalent Fractions & Decimals
- Using ratios to express parts of whole quantities
- Expressing ratios in decimal form
- Expressing ratios as percents

Session 3 - Forming Ratios between Unlike Quantities
- Forming ratios by comparing different quantities
- Using a pie chart (circle graph) to represent ratios

Unit 2 - Proportion

Session 1 - Defining a Proportion
- Recognizing a proportion as an equivalence between ratios
- Writing equivalent ratios as equivalent fraction

Session 2 - Solving for a Variable in a Proportion
- Setting up a proportion involving a variable
- Solving for the variable in a proportion
- Recognizing the means/extremes property: if a:b = c:d, then ad = bc
- Identifying the means and the extremes in a proportion

Session 3 - Applying the Means/Extremes Property
- Solving for the variable in a proportion using cross-multiplication
- Calculating cross-products to check a solution in a proportion
- Converting standard units to metric units using proportions

Unit 3 - Direct & Inverse Variation

Session 1 - Exploring & Solving Direct Variation Problems
- Recognizing a direct variation
- Using the symbol for proportion to represent a direct variation
- Expressing a direct variation as a proportion
- Solving a proportion for a variable

Session 2 - Exploring Inverse Variation
- Recognizing an inverse variation
- Using the symbol for proportion to represent an inverse relationship
- Expressing an inverse relationship as a proportion
- Writing an inverse variation as two equivalent products

Session 3 - Solving Inverse Variation Problems
- Solving an inverse relationship for a missing quantity
- Comparing an inverse variation to a direct variation

Unit 4 - Similar Polygons

Session 1 - Defining Similarity
- Recognizing the meaning of similarity
- Writing a proportion that can be used to solve for a variable

Session 2 - Identifying Equivalent Ratios
- Applying the definition of similarity to identify equivalent ratios
- Identifying corresponding sides in similar polygons
- Using similarity to set up proportions involving corresponding sides
- Defining 'polygon'

Session 3 - Setting up & Solving Proportions in Similar Polygons
- Recognizing a right triangle
- Applying the Pythagorean Theorem to find the third side of a right triangle
- Setting up and solving equations based on ratios between corresponding sides
- Using scaling to determine corresponding lengths in similar polygons

Module 5 - Fundamentals of Statistics

Unit 1 - Interpreting and Constructing Graphs

Session 1 - Exploring Line Graphs
- Interpreting a line graph
- Adding points to a line graph
- Identifying increasing and decreasing trends on a line graph

Session 2 - Exploring Bar Graphs
- Interpreting a bar graph
- Identifying data sets
- Identifying the horizontal and vertical axes
- Identifying the range of a data set
- Creating a scale along an axis
- Constructing a bar graph
- Using a broken axis to scale data

Session 3 - Interpreting Pie Charts
- Interpreting a pie chart
- Converting raw data to percents
- Finding the number of degrees in a sector
- Creating a sector using a protractor
- Finding the number of degrees in a sector
- Constructing a pie chart

Unit 2 - The Mean, Median, & Mode

Session 1 - Defining the Mean & Median
- Defining raw data
- Defining a sample
- Naming the three measures of central tendency
- Defining the mean
- Defining the median

Session 2 - Defining the Mode
- Defining the mode
- Interpreting which measure best represents the 'average' for a given set of data

Unit 2 - The Mean, Median, & Mode (cont'd)

Session 3 - Calculating the Mean, Median, & Mode

- Calculating the mean
- Calculating the median
- Determining the mode
- Interpreting which measure best represents the 'average' for a given set of data

Unit 3 - Frequency Distribution and Histograms

Session 1 - Creating & Interpreting a Frequency Table

- Using tally marks to create a frequency table
- Constructing a frequency distribution
- Calculating the mean using the frequency data

Session 2 - Defining a Histogram

- Dividing data into equal intervals to create a grouped frequency table
- Defining a histogram
- Creating a histogram for the frequency data
- Finding the mean of a grouped frequency

Session 3 - Exploring Cumulative Frequency Graphs

- Calculating and plotting cumulative frequencies on a graph
- Identifying a best-fit curve for the points on a cumulative frequency graph
- Finding a specified percentile using a cumulative frequency graph

Module 6 - Fundamentals of Probability

Unit 1 - Simple Probability

Session 1 - Defining & Expressing Probability

- Defining the probability of an outcome in an experiment
- Recognizing that the sum of the probabilities of all possible outcomes in an experiment is 1
- Recognizing that the probability of an impossible outcome is 0
- Defining the sample space for an experiment
- Expressing probabilities as fractions and percents

Session 2 - Calculating Probabilities on a Color Wheel

- Determining the sample space on a color wheel
- Calculating the probabilities of different outcomes when spinning a color wheel

Session 3 - Determining Probabilities of Complementary Events

- Calculating the probabilities of different outcomes when spinning a color wheel

Unit 2 - Probability of Combined Events

Session 1 - Calculating the Probability of Independent Events

- Identifying independent events
- Determining the sample space of an experiment using a table
- Calculating the probability of an event
- Calculating the probability of independent events

Session 2 - Determining the Sample Space of an Experiment

- Determining the probability of a certainty
- Recognizing mutually exclusive events
- Determining the sample space of an experiment using a table

Session 3 - Calculating the Probability of Mutually Exclusive Events

- Using a tree diagram to determine probabilities
- Identifying dependent events
- Calculating the probability of mutually exclusive events
- Verifying the probability formulas using a tree diagram

Mastering Algebra I: Course A

Skill Levels: Grades 8-10
83 Learning Objectives

In this course, students focus on the symbols and rules of algebra and how they are used to represent relationships. Students use these concepts to solve linear equations in one variable and apply these skills to real-life problems. They progress through the course by graphing linear functions and systems, solving the latter both graphically and algebraically. A study of linear inequalities in one and two variables parallels the study of linear equalities with an exploration of absolute value.

Module 1 - The Language of Algebra

Unit 1 - Variables, Expressions, and Equations

Session 1 - Translating Words into Expressions
- Recognizing the various representations of an algebraic relationship
- Identifying the meaning of a variable in a given situation
- Writing equivalent algebraic expressions for verbal phrases

Session 2 - Applying Properties of Real Numbers
- Applying the commutative properties of addition and multiplication
- Applying the associative properties of addition and multiplication
- Applying the distributive property of multiplication over addition

Session 3 - Evaluating and Simplifying Expressions
- Combining like terms using the properties of real numbers
- Evaluating expressions and formulas for given values of one or more variables

Unit 2 - Linear Equations in One Variable

Session 1 - Applying Inverse Operations
- Exploring the addition, subtraction, multiplication, and division properties of equality
- Applying the addition and subtraction properties of equality to solve one-step equations
- Verifying by substitution that the solution to an equation is valid
- Applying the multiplication and division properties of equality to solve one-step equations
- Rewriting formulas for specific variables

Session 2 - Transforming Equations using Multiple Operations
- Solving equations of the form $ax + b = c$
- Solving equations of the form $a(x + b) = c(x + d)$

Session 3 - Solving Absolute Value Equations
- Exploring the meaning of the geometric definition of absolute value
- Applying the geometric definition of absolute value to solve absolute value equations
- Exploring the meaning of the algebraic definition of absolute value
- Applying the algebraic definition of absolute value to solve absolute value equations

Module 2 -Linear Functions and Equations

Unit 1 - The Rectangular Coordinate Plane

Session 1 - Graphing Ordered Pairs
- Reading the coordinates of a point from a graph
- Creating valid scales to graph sets of data
- Recognizing types of correlation given a set of ordered pairs

Session 2 - Defining Slope
- Determining the rise and run between a pair of points
- Defining the slope, m, as the ratio of rise over run
- Calculating the slope of a non-vertical line given the coordinates of any two points on the line
- Recognizing how the sign of the slope of a line determines how the line lies in a plane
- Determining the slope of a horizontal line
- Examining why vertical lines have undefined slopes

Session 3 - Finding x- and y- Intercepts
- Identifying the horizontal and vertical intercepts of a line
- Investigating the meaning of the x- and y- intercepts
- Determining if three or more points are collinear
- Interpreting the meaning of a broken line graph

Unit 2 - Introduction to Functions

Session 1 - Exploring the Slope-Intercept Equation of a Line
- Expressing the relationship between x and y as an equation given a table of values, $b = 0$
- Recognizing that the value of the slope of a non-vertical line is the coefficient of x in the equation $y = mx$
- Writing the equation of a line given its graph through the origin and the coordinates of a second point on the line
- Recognizing the value of the y-intercept of a line as the constant b in the equation $y = mx + b$
- Graphing a line given its equation in the form $y = mx + b$, where b does not equal zero
- Writing the equation of a line in slope-intercept form given the graph of a non-vertical line and the coordinates of the y-intercept and a second point on the line

Session 2 - Exploring the Point-Slope Equation of a Line
- Finding the coordinates of a point on a line given the slope and the coordinates of one point on the line
- Finding the value of the y-intercept given the slope of a line and the coordinates of a point on the line
- Using the definition of the slope of a non-vertical line, express the equation of a line in the form $(y - y1) = m(x - x1)$
- Identifying the slope and the coordinates of a point on the line given an equation of the form $(y - y1) = m(x - x1)$

Session 3 - Relations and Functions
- Defining a function
- Defining the domain and range of a function
- Expressing equations of lines as functions
- Evaluating f(x) for a given function f and values of x
- Analyzing the domain and range of the absolute value function
- Defining a relation

Module 3 - Systems of Linear Equations

Unit 1 - Graphic Solutions of Linear Systems

Session 1 - Finding the Point of Intersection

- Solving a linear system by finding the coordinates of the point of intersection of the graphs of the system
- Verifying by substitution that the coordinates of the point of intersection of two non-vertical lines satisfy the equations of each line
- Recognizing that the graph of a system of linear equations is not an image of the real-life situation it represents
- Solving an equation in one variable by expressing each side as a function and graphing the system

Session 2 - Graphing Parallel & Perpendicular Lines

- Verifying that the slopes of perpendicular lines are negative reciprocals
- Confirming that if the product of the slopes to two non-vertical lines is -1, the lines are perpendicular
- Verifying that if two non-vertical lines are parallel, their slopes are equal
- Confirming that if the slopes of two lines are equal, the lines are parallel
- Justifying by graphing that a linear system consisting of parallel lines has no solution

Unit 2 - Algebraic Solutions of Linear Systems

Session 1 - Using Substitution to Eliminate a Variable

- Using substitution to eliminate one variable when both equations of the system are expressed in terms of one of the two variables
- Using substitution to eliminate one variable when one or both equations of the system are not expressed in terms of one of the two variables
- Recognizing that the solution (k, q) of a linear system lies on the lines $x = k$ and $y = q$

Session 2 - Using Addition or Subtraction to Eliminate a Variable

- Using addition and subtraction to eliminate one variable in a system of equations
- Using multiplication and addition or subtraction to eliminate one variable in a system of equations

Module 4 - Linear Inequalities

Unit 1 - Inequalities in One Variable

Session 1 - Applying Inverse Operations

- Isolating the variable in an inequality involving one variable using the operations of addition and subtraction
- Applying the rule regarding sign change when multiplying or dividing by a negative number
- Using two or more transformations to solve an inequality

Session 2 - Graphing Solutions on a Number Line

- Graphing a simple inequality on a number line
- Investigating multiple representations of the intersection of inequalities
- Investigating multiple representations of the union of inequalities
- Writing an algebraic expression for the graph of a compound inequality

Session 3 - Solving Absolute Value Inequalities

- Writing a compound inequality as an absolute value inequality
- Representing the graph of a compound inequality as an absolute value inequality
- Identifying the complement of a given set
- Graphing the solution set of an absolute value inequality on a number line
- Applying the algebraic definition of absolute value to solve an absolute value inequality

Unit 2 - Inequalities in Two Variables

Session 1 - Graphing Solutions on a Rectangular Coordinate Plane

- Defining half-planes and boundary lines
- Identifying the relationship between the ordered pairs in a half-plane
- Locating a point in a given half-plane
- Graphing a linear inequality

Session 2 - Solving Systems by Graphing

- Solving a system defined by two inequalities
- Defining the reasonableness of a solution for a given situation
- Writing a system of inequalities that describes the constraints of a linear programming problem
- Identifying the feasible region of a linear programming problem
- Identifying the maximum/minimum values of the solution of a linear programming problem as the coordinates of the vertices of the feasible region

Mastering Algebra I: Course B

Skill Levels: Grades 8-10
87 Learning Objectives

In this course, students investigate polynomial expressions and operations in problems that arise from real-life situations. Students refine and expand their skills through interactions, practice problems, and workout questions. With these skills in hand, they graph parabolas and identify relationships between graphs and equations. Following this analytic investigation, students use a variety of techniques to solve equations in one variable, including applying the quadratic formula.

Module 1 - The Real Number System

Unit 1 - Rational & Irrational Numbers

Session 1 - Defining the Real Numbers
- Defining rational numbers
- Defining irrational numbers
- Using the Pythagorean Theorem to demonstrate the existence of irrational numbers
- Approximating the square roots of a set of real numbers and locating them on a number line

Session 2 - Working with Radicals
- Evaluating the square root of a perfect square
- Simplifying the square root of a product
- Simplifying the quotient of two radicals
- Rationalizing the denominator of a radical expression
- Adding or subtracting radical expressions using the distributive property

Session 3 - The Square Root Function
- Graphing a finite set of ordered pairs (x, sqrt(x))
- Graphing the square root function
- Identifying the domain, range, and equation of the square root function
- Examining the effect of a on the graph of y = a * sqrt(x)

Module 2 - Powers & Polynomials

Unit 1 - Polynomial Arithmetic

Session 1 - Working with Powers
- Simplifying expressions containing negative exponents and 0
- Simplifying expressions involving the product and quotient of two powers
- Simplifying expressions involving the power of a power
- Simplifying expressions involving the power of a product and a quotient

Session 2 - Adding & Subtracting Polynomial Expressions
- Exploring the definitions related to polynomial expressions
- Arranging the terms of a polynomial expression in ascending or descending order
- Finding the sum and difference of two (or more) polynomials

Session 3 - Multiplying Polynomials
- Using an area model to represent the product of two binomials
- Using the distributive property to find the product of two polynomials
- Recognizing the square of a binomial as a perfect square trinomial
- Recognizing the product of the sum and difference of two monomials as the difference of two squares

Unit 2 - Factoring Polynomials

Session 1 - Finding Common Factors
- Discriminating between prime and composite numbers
- Identifying the greatest common monomial factor of two (or more) monomials
- Factoring a polynomial by finding its greatest common monomial factor
- Factoring a polynomial by finding a common binomial factor

Session 2 - Factoring Quadratic Trinomials
- Factoring a quadratic trinomial of the form $1x^2 + bx + c$, where $c > 0$
- Factoring a quadratic trinomial of the form $1x^2 + bx + c$, where $c < 0$
- Factoring a quadratic trinomial of the form $ax^2 + bx + c$, where a is not equal to 1

Session 3 - Special Cases
- Recognizing and factoring a perfect square trinomial: $a^2 \pm 2ab + b^2$
- Recognizing and factoring the difference of two squares: $a^2 - b^2$
- Factoring a given polynomial completely

Module 3 - Quadratic Functions & Equations

Unit 1 - Graphing Quadratic Functions & Equations

Session 1 - Graphing Parabolas
- Recognizing that the graph of the quadratic equation $y = ax^2$ is a function
- Identifying the domain and range of the parabola whose equation is $y = ax^2$
- Describing the effect of the parameter a on the shape of the graph of the function $y = ax^2$
- Determining the minimum and maximum of a parabolic function of the form $y = ax^2$
- Determining the equation of the axis of symmetry of a parabola of the form $y = ax^2$
- Determining the coordinates of the vertex of a parabola of the form $y = ax^2$

Session 2 - Analyzing Properties of Parabolas
- Examining the properties of parabolas whose equations are of the form y = ax^2 + c, where c is not equal to 0
- Recognizing that the constant c in a quadratic function y = ax^2 + bx + c is the y-intercept of a parabola
- Examining the properties of parabolas whose equations are of the form y = ax^2 + bx
- Examining the properties of parabolas whose equations are of the form y = ax^2 + bx + c, where b is not equal to 0, and c is not equal to 0

Session 3 - Solving Quadratic Equations by Graphing
- Recognizing that if a parabola $y = ax^2 + bx + c$ has two x-intercepts, there are two real solutions to the corresponding quadratic equation, $ax^2 + bx + c = 0$
- Discovering that the maximum number of real solutions of a quadratic equation is 2
- Recognizing that if a parabola has only one x-intercept, there is only one real solution to the corresponding quadratic equation, $ax^2 + bx + c = 0$
- Recognizing that if a parabola does not intersect the x-axis, the corresponding quadratic equation, $ax^2 + bx + c = 0$, has no real solution

Unit 2 - Solving Quadratic Equations Using Algebra

Session 1 - Factoring & the Zero Product Theorem

- Recognizing that the solutions of a quadratic equation are the x-intercepts of the corresponding function
- Solving a quadratic equation in one variable by factoring the difference of two squares
- Solving a complete quadratic equation in one variable by factoring
- Solving a quadratic equation in one variable by factoring a perfect square trinomial

Session 2 - The Square Root Method & Completing the Square

- Finding the real roots of a quadratic equation using the square root property
- Finding the rational roots of a quadratic equation by completing the square
- Finding the irrational roots of a quadratic equation by completing the square

Session 3 - The Quadratic Formula

- Recognizing the steps in the proof of the quadratic formula and interpreting its meaning
- Finding the real roots of a quadratic equation using the quadratic formula
- Using the quadratic formula to determine that a quadratic equation does not have real roots
- Using the discriminant to determine the nature of the roots of a quadratic equation in one variable

Module 4 - Algebraic Expressions & Functions

Unit 1 - Radical Equations & Functions

Session 1 - Solving Radical Equations

- Recognizing and solving a simple radical equation
- Determining if a radical equation has a real solution
- Solving radical equations algebraically
- Determining if a radical equation has an extraneous solution

Session 2 - The Inverse of the Square Root Function

- Graphing the inverse of the square root function and identifying its equation
- Calculating the equation of the line of symmetry between the square root function and its inverse
- Examining the inverse of a parabolic function with a restricted domain

Unit 2 - Rational Expressions, Equations & Functions

Session 1 - Rational Operations

- Identifying value(s) of x for which a rational expression is undefined
- Reducing a rational expression by removing common monomial factors
- Finding the product or quotient of two rational expressions and expressing it in simplest form
- Identifying the least common denominator of two algebraic fractions
- Finding the sum or difference of two rational expressions and expressing it in simplest form

Session 2 - Rational Functions

- Graphing the function $f(x) = 1/x$
- Identifying the domain and range of $f(x) = 1/x$
- Identifying the equations of the asymptotes and the inverse of $f(x) = 1/x$
- Examining the effect of parameters a and b on the graph of $f(x) = a/(x - b)$

Session 3 - Rational Equations

- Solving a rational equation by multiplying by the LCD
- Analyzing and solving a work problem
- Analyzing and solving a uniform motion problem
- Determining if a solution of a rational equation is extraneous

Module 5 - Describing Data

Unit 1 - Graphical Displays

Session 1 - Stem-and-Leaf Plots & Box Plots

- Creating and analyzing a stem-and-leaf plot
- Calculating the range and median of a set of data
- Creating a box plot
- Analyzing the information in a box plot

Session 2 - Scatter Plots & Linear Best-Fit Graphs

- Approximating a line of best-fit through a set of points in a scatter plot
- Calculating and interpreting the median-median best-fit line for a set of data in a scatter plot
- Using a best-fit line to predict a future value
- Comparing actual and predicted values using a best-fit line

NCTM Content Standards
NCTM Process Standards

Scope & Sequence Chart

Course & Module	Module Description	Numbers and Operations	Algebra	Geometry	Measurement	Data Analysis and Probability	Problem Solving	Reasoning & Proof	Communication	Connections	Representation
Mastering Skills & Concepts I: Pre-Primary Mathematics | *Grades K–1*											
Number Sense	Explore numbers from 0 to 100 in many ways: represent, make sets, count, skip-count, compare, and order.	•					•		•		•
Addition and Subtraction	Find sums and differences within 20.	•					•			•	•
Geometry and Measurement	Explore measurement basics, focusing on ordinal numbers, length and weight, clock and calendar, and money; and, in geometry, basics of triangles, rectangles, and 3-D shapes.	•		•	•		•				•
Algebraic Thinking	Investigate patterns: linear patterns involving shapes, as well as number patterns; and represent and analyze data using picture graphs, frequency tables, and bar graphs.	•		•		•	•		•		•
Mastering Skills & Concepts II: Primary Mathematics | *Grades 2–3*											
Number Sense	Explore numbers to 1,000: find place value to thousands, compare and order.	•					•				•
Operations with Numbers	Estimate and find sums less than 1,000, differences within 9,999; find products less than 100; divide by 1-digit number; find fractional parts.	•		•			•			•	•
Geometry and Measurement	Determine area and volume in standard units; expand skills with time, money (making change); temperature.			•	•		•		•	•	•
Algebraic Thinking	Learn commutative and associative properties of addition and multiplication; investigate linear patterns between two quantities.	•	•				•		•	•	•
Mastering Skills & Concepts III: Intermediate Mathematics | *Grades 4–6*											
Numbers and Number Sense	Explore whole numbers to one million, including negative numbers, factors, and prime numbers.	•	•	•			•		•	•	•
Operations with Numbers	Understand and perform addition and subtraction of large numbers; two-digit multiplication; long division using two-digit divisors.	•		•			•	•		•	•
Fractions	Introduce proper and improper fractions, ordering fractions; addition and subtraction with like and unlike denominators; multiplying and dividing fractions.	•					•	•			•
Decimals	Explore decimals, including expressing, ordering, rounding, and converting decimals to thousandths; adding, subtracting, multiplying, and dividing decimals.	•				•	•	•		•	•
Geometry	Define and explore relationships among lines, segments, rays, and angles to classify and measure area of two-dimensional shapes; investigate coordinate geometry, symmetry, and transformations.		•	•	•		•	•	•		•
Data Analysis and Probability	Develop skills at modeling, displaying, and analyzing data; determining and representing probability.					•	•			•	•

■ NCTM Content Standards
■ NCTM Process Standards

Scope & Sequence Chart

Course & Module	Module Description	Numbers and Operations	Algebra	Geometry	Measurement	Data Analysis and Probability	Problem Solving	Reasoning & Proof	Communication	Connections	Representation
Mastering Skills & Concepts IV: Basic Mathematics | *Grades 6–8*											
Fractions	Explore multiple aspects of fractions, beginning with mixed numbers, equivalent fractions, and reducing fractions; multiply, divide, add, and subtract fractions.	•		•	•		•			•	•
Decimals	Explore multiple aspects of decimals, beginning with rounding decimals, including repeating and terminating decimals; add, subtract, multiply, and divide decimals, including estimating and finding quotients.	•		•	•		•			•	•
Percents	Understand relationships between percents and fractions, ratios, parts and whole; calculate percents.	•				•	•			•	•
Integers and Order of Operations	Understand and apply absolute value; add, subtract, multiply, and divide signed numbers; learn order of operations.	•			•		•		•	•	
Mastering Skills & Concepts V: Pre-Algebra | *Grades 6–8*											
Essentials of Algebra	Begin to understand variables; evaluate, simplify, and solve simple equations; solve equations with variables on both sides; and identify variables.	•	•	•	•		•	•		•	•
Fundamentals of Geometry	In two-dimensional geometry, focus on triangles: understanding complementary, supplementary, and congruent angles; and classifying triangles. In three-dimensional geometry, focus on volume and surface area, specifically of prism and of right cylinder.		•	•	•		•	•	•	•	•
Radicals and Exponents	Explore variety of concepts such as radicals, Pythagorean Theorem, and Scientific Notation; apply to concepts and applications such as squares, square roots, and irrational numbers.	•	•	•	•		•	•		•	•
Ratio and Proportion	Explore ratio and proportional relationships, using equivalent fractions, variables, and inverse variation; apply ratio and proportion to analyzing polygons.	•	•	•	•		•			•	
Fundamentals of Statistics	Display and interpret data visually and using various types of averages; specifically, use line graphs, bar graphs, pie charts, frequency tables, and histograms; define and calculate mean, median, and mode.		•		•	•	•			•	•
Fundamentals of Probability	Determine and express probability of various types, from simple probability to probability of complementary events to probability of combined events.					•	•			•	
Mastering Algebra I: Course A | *Grades 8–10*											
The Language of Algebra	Understand, create, simplify, evaluate expressions and formulas, using single-variable equations with one step and with multiple operations; explore absolute value.	•	•	•	•		•			•	•
Linear Functions and Equations	Explore and graph on rectangular coordinate plane; with slope-intercept equation and point-slope equation as introduction to functions.		•	•		•	•	•		•	•
Systems of Linear Equations	Graph lines to represent given data sets to solve the corresponding system of equations, including interpreting line slopes; explore algebraic solutions to solve systems of multiple linear equations.		•			•	•			•	•
Linear Inequalities	Solve inequalities algebraically by isolating the variable, applying rules about negative number operations, and graph inequality relationships; also absolute-value inequalities, and graphing inequalities in two variables.	•	•	•			•	•	•	•	

NCTM Content Standards
NCTM Process Standards

Scope & Sequence Chart

Course & Module	Module Description	Numbers and Operations	Algebra	Geometry	Measurement	Data Analysis and Probability	Problem Solving	Reasoning & Proof	Communication	Connections	Representation
Mastering Algebra I: Course B | *Grades 8–10*											
The Real Number System	Focus on rational and irrational numbers, including radicals and square roots, including square-root functions and graphs.	•	•	•			•	•		•	•
Powers and Polynomials	Focus on polynomial expressions, specifically simplifying, comparing, adding, multiplying, and dividing polynomials.	•	•	•			•			•	•
Quadratic Functions	Graph quadratic functions and equations; solve quadratic equations using algebra, including the quadratic formula.	•	•	•			•	•		•	•
Algebraic Expressions	Recognize and solve radical equations and functions, including determining if an equation has a solution and ways of graphing and solving such equations.		•				•	•		•	•
Describing Data	Create and analyze stem-and-leaf plots, box plots, and scatter plots, including using best-fit lines.					•	•			•	•

DESTINATION
MATH

hmheducation