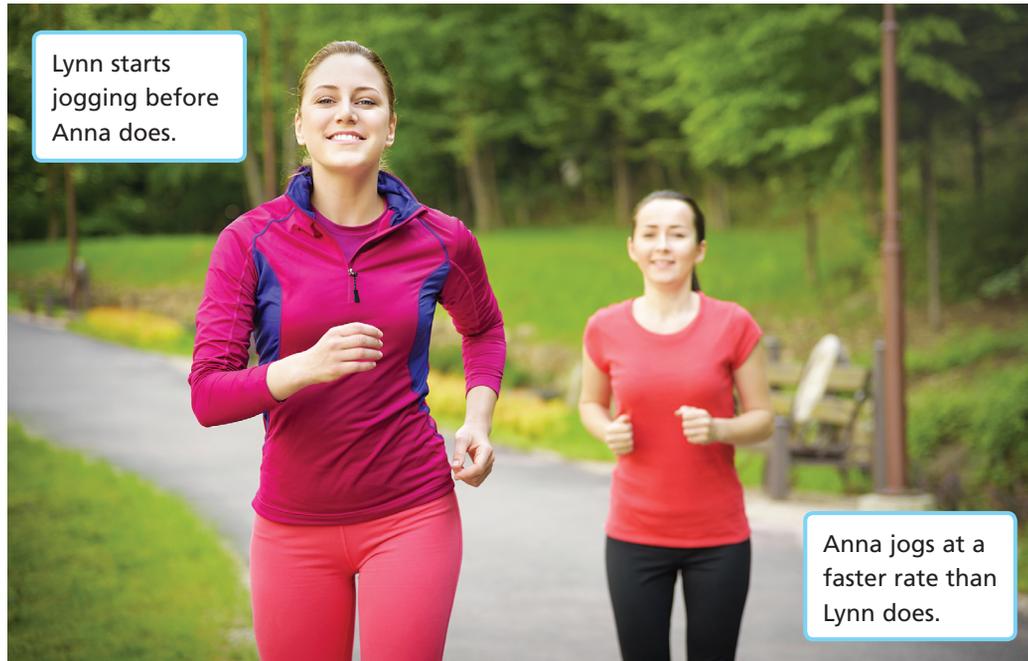


Write and Solve Equations

I Can use the properties of equality and the Distributive Property to solve an equation with the variable on both sides.

Spark Your Learning

Lynn and Anna are joggers who use the same running trail.



Complete Part A as an entire class. Then complete Parts B–D in small groups.

- What mathematical question can you ask about this situation? What information would you need to know to answer your question?
- What variable(s) are involved in this situation? What unit of measurement would you use for each variable?
- How would you use the information in the photo and the additional information your teacher gave you to answer your question? What is the answer?
- Does your answer make sense in the context of the situation? How do you know?



Turn and Talk Predict how your answer would change for each of the following changes in the situation:

- Anna starts jogging 3 minutes earlier.
- Lynn's average jogging speed is 0.2 mile per hour faster.
- Lynn and Anna jog at the same average speed.

Build Understanding

Investigate Properties of Equality

An **equation** is a mathematical statement comparing two expressions using the symbol $=$. When you solve an equation, the **solution** produces a true statement when substituted for the variable in the equation. Solving an equation involves using properties of equality to write simpler **equivalent equations**, which all have the same solution as the original equation. In the Symbols column of the table below, a , b , and c are real numbers.

Properties of Equality		
Property	Words	Symbols
Addition Property of Equality	Adding the same number to both sides of an equation produces an equivalent equation.	If $a = b$, then $a + c = b + c$. Example: If $x - 2 = 3$, then $x - 2 + 2 = 3 + 2$.
Subtraction Property of Equality	Subtracting the same number from both sides of an equation produces an equivalent equation.	If $a = b$, then $a - c = b - c$. Example: If $x + 4 = -1$, then $x + 4 - 4 = -1 - 4$.
Multiplication Property of Equality	Multiplying both sides of an equation by the same nonzero number produces an equivalent equation.	If $a = b$ and $c \neq 0$, then $ac = bc$. Example: If $\frac{x}{3} = 2$, then $\frac{x}{3} \cdot 3 = 2 \cdot 3$.
Division Property of Equality	Dividing both sides of an equation by the same nonzero number produces an equivalent equation.	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. Example: If $-2.5x = 10$, then $\frac{-2.5x}{-2.5} = \frac{10}{-2.5}$.

- 1 A.** Using a spreadsheet, create a table like the one shown for the equation $x = -1$. The formula in cell B2 is an if-then-else statement that checks the value in cell A2. Fill down the formula from cell B2 to cell B8. What do you observe?

B2 $\times \checkmark f_x$ = IF(A2 = -1, "YES", "NO")					
	A	B	C	D	E
1	x	Is x a solution?			
2	-3	NO			
3	-2				
4	-1				
5	0				
6	1				
7	2				
8	3				

If the number in cell A2 equals -1, then the word YES is shown in cell B2. Otherwise, the word NO is shown in cell B2.

- B.** Suppose you use the Multiplication Property of Equality to rewrite the equation $x = -1$ as $2x = -2$. Use the new equation in if-then-else statements you enter in cells C2 through C8. What do you observe?
- C.** Suppose you use the Addition Property of Equality to rewrite the equation $2x = -2$ as $2x + 3 = 1$. Use the new equation in if-then-else statements you enter in cells D2 through D8. What do you observe?



Turn and Talk You used properties of equality to build the equation $2x + 3 = 1$ from $x = -1$. How can you use properties of equality to solve the equation $2x + 3 = 1$?

Step It Out

Solve Equations Using the Distributive Property

Previously, you solved one-step and two-step equations. Now you will solve multistep equations. Such equations may contain grouping symbols. In order to free the terms in an equation that contains grouping symbols, you can use the Distributive Property.

2 The steps for solving the equation $5(2x - 3) + 4 = -6$ and checking the solution are shown below, but the steps have been scrambled.

A. Write the solution steps in the correct order.

$$10x - 11 = -6$$

$$x = 0.5$$

$$10x - 15 + 4 = -6$$

$$5(2x - 3) + 4 = -6$$

$$10x = 5$$

B. Write the check steps in the correct order.

$$-6 = -6 \quad \checkmark$$

$$5(2(0.5) - 3) + 4 \stackrel{?}{=} -6$$

$$5(-2) + 4 \stackrel{?}{=} -6$$

$$5(1 - 3) + 4 \stackrel{?}{=} -6$$

$$-10 + 4 \stackrel{?}{=} -6$$



Turn and Talk Can you solve the equation $5(2x - 3) + 4 = -6$ *without* using the Distributive Property as one of the steps? Show how or explain why not.

Recall that the Distributive Property also allows you to combine like terms.

3 The steps for solving the equation $-2x + \frac{1}{2}(6x - 5) = \frac{3}{2}$ are shown, but some justifications are missing.

$$-2x + \frac{1}{2}(6x - 5) = \frac{3}{2}$$

Given equation

A. What property justifies rewriting $\frac{1}{2}(6x - 5)$ as $3x - \frac{5}{2}$?

$$-2x + 3x - \frac{5}{2} = \frac{3}{2}$$

?

B. What property justifies rewriting $-2x + 3x$ as $(-2 + 3)x$?

$$(-2 + 3)x - \frac{5}{2} = \frac{3}{2}$$

?

$$x - \frac{5}{2} = \frac{3}{2}$$

Simplify the coefficient of x .

C. What property justifies adding $\frac{5}{2}$ to each side of the equation?

$$x = 4$$

?



Turn and Talk Is it possible to eliminate the fractions as a first step in solving the equation $-2x + \frac{1}{2}(6x - 5) = \frac{3}{2}$? Show how or explain why not.

Solve Equations with the Variable on Both Sides

When a variable appears on both sides of an equation, you can use the Addition Property of Equality or the Subtraction Property of Equality to move the variable from one side to the other. Doing so allows you to isolate the variable on one side.

- 4** The steps for solving the equation $4(x + 1) + 1 = -3(x + 3)$ are shown, but some justifications are missing.

$4(x + 1) + 1 = -3(x + 3)$	Given equation	A. What property lets you rewrite each side without grouping symbols?
$4x + 4 + 1 = -3x - 9$	<u>?</u>	
$4x + 5 = -3x - 9$	Combine constants.	B. What property justifies adding $3x$ to each side?
$4x + 5 + 3x = -3x - 9 + 3x$	<u>?</u>	
$7x + 5 = -9$	Combine like terms.	C. What property justifies subtracting 5 from each side?
$7x = -14$	<u>?</u>	
$x = -2$	Division Property of Equality	



Turn and Talk How is solving $4(x + 1) + 1 = 4(x + 3)$ different from solving $4(x + 1) + 1 = -3(x + 3)$? What is different about solving $4(x + 1) + 8 = 4(x + 3)$?

Use an Equation to Solve a Real-World Problem

When solving a real-world problem, you may have to use a formula, such as the formula $P = 2\ell + 2w$ for the perimeter of a rectangle or the formula $d = rt$ for the distance traveled at a constant rate. When using a formula, you should pay attention to the units of measurement associated with the variables to ensure that the units are consistent.

Perimeter ?	= 2 •	Length (meters)	+ 2 •	Width ?
----------------	-------	--------------------	-------	------------

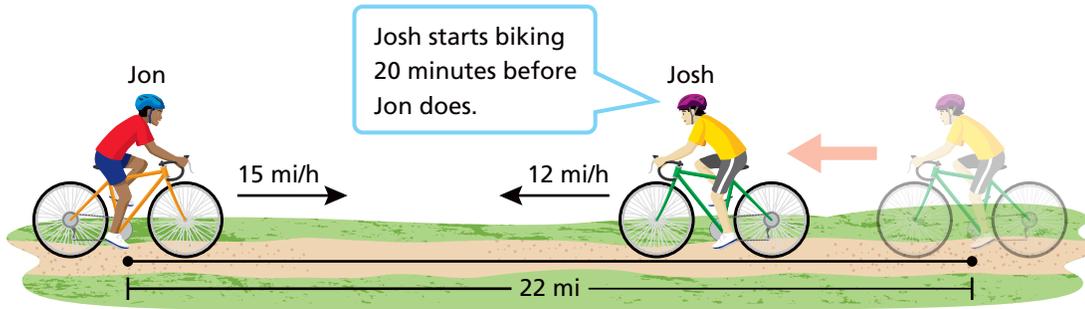
Since the length is in meters, the **width** must also be in **meters** so you can add the units on the right side. This means the **perimeter** is in **meters** because meters + meters = meters.

Distance (feet)	=	Rate ?	•	Time (seconds)
--------------------	---	-----------	---	-------------------

For the units to be consistent in this equation, the rate must be measured in *feet per second* and not, say, *miles per hour*.

5

Two friends, Jon and Josh, live at opposite ends of a trail. They bike toward each other at the speeds shown. At what distance along the trail, measured from Jon's starting point, do the two friends meet?

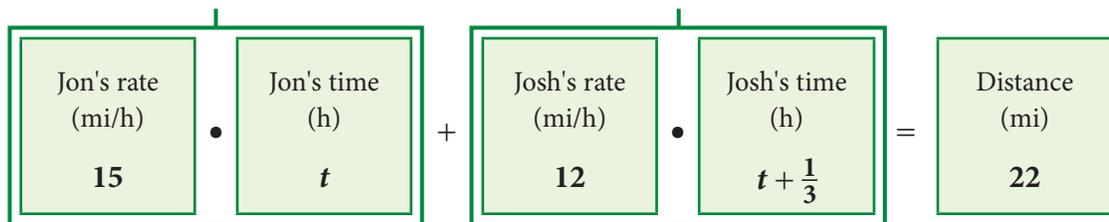


Use a verbal model.

Although you are asked to find distance, you will do so by expressing distance in terms of time. Note that the units *miles per hour* and *minutes* are incompatible. So, let t represent the time in *hours* that Jon spends biking.

Jon's distance (mi)

Josh's distance (mi)



Write an equation.

$$15t + 12\left(t + \frac{1}{3}\right) = 22$$

Solve the equation.

$$15t + 12\left(t + \frac{1}{3}\right) = 22$$

Given equation

$$15t + 12t + 4 = 22$$

Distributive Property

$$27t + 4 = 22$$

Combine like terms.

$$27t = 18$$

Subtraction Property of Equality

$$t = \frac{2}{3}$$

Division Property of Equality

Answer the question.

The two friends will meet at a distance of $15\left(\frac{2}{3}\right) = 10$ miles from Jon's starting point on the trail.

A. Why is this expression $t + \frac{1}{3}$ and not $t + 20$?

B. How can you use the solution of the equation to find the distance from Jon's starting point? What is this distance?



Turn and Talk Suppose the trail in Task 5 is only 3 miles long. Write and solve a new equation to find the time Jon spends biking. Does your solution make sense? Explain.

Check Understanding

1. Which two of the equations shown below are equivalent? Explain your reasoning.

$$2x + 7 = 1 \qquad -3x + 4 = 12 \qquad x = -3$$

Solve each equation. Justify your solution steps, and check each solution.

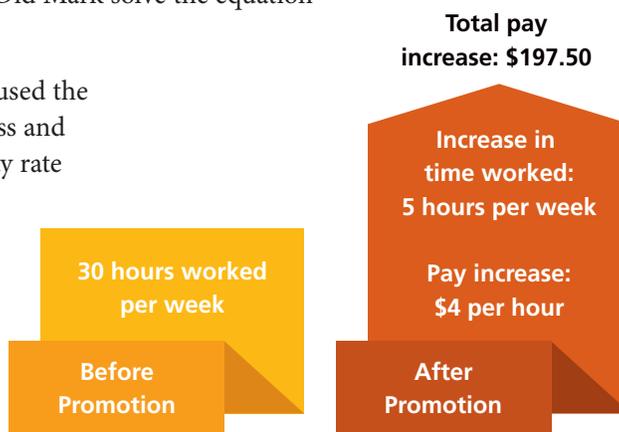
2. $6(2x - 5) - 8 = 4$ 3. $\frac{1}{2}(4x - 3) = \frac{3}{4}(4x - 5)$
4. Membership at the local art center is \$60 per year. If you are a member, you can take lessons at the center for \$7 each. If you are not a member, lessons cost \$10 each. For how many lessons is the total cost with a membership the same as the total cost of lessons without a membership?

On Your Own

5. **(MP) Critique Reasoning** Mark solved the equation $\frac{2}{3}x = 9$ by using the Division Property of Equality to get $x = 6$. Did Mark solve the equation correctly? Explain why or why not.

6. **(MP) Use Repeated Reasoning** Melissa used the information shown at the right and the guess and check method to find the manager's new pay rate after being promoted.

- A. Melissa's work is shown in the table below. Explain how she can generalize what she has done by defining a variable and then writing and solving an equation using that variable.



New pay rate guess	Calculated new weekly pay	Calculated old weekly pay	Calculated increase in weekly pay
\$12.00 per hour	$35(\$12.00) = \420.00	$30(\$12.00 - \$4.00) = \$240.00$	$\$420.00 - \$240.00 = \$180.00$ ✗
\$15.00 per hour	$35(\$15.00) = \525.00	$30(\$15.00 - \$4.00) = \$330.00$	$\$525.00 - \$330.00 = \$195.00$ ✗
\$16.00 per hour	$35(\$16.00) = \560.00	$30(\$16.00 - \$4.00) = \$360.00$	$\$560.00 - \$360.00 = \$200.00$ ✗
\$15.50 per hour	$35(\$15.50) = \542.50	$30(\$15.50 - \$4.00) = \$345.00$	$\$542.50 - \$345.00 = \$197.50$ ✓

- B. Suppose the question had been, "What was the manager's old pay rate?" Define the variable in a way that will answer this question. Then write and solve a new equation to show that you get a solution that is consistent with the one from Part A.

Solve each equation. Justify your solution steps, and check each solution.

7. $4(3x - 10) + 7 = 15$

8. $6 - 5(2x + 1) = 21$

9. $2 - \frac{3}{4}(8x - 6) = 11$

10. $0.2(4 - 5x) + 1 = 2.4$

11. $5(x + 1) - 2(3x - 4) = 14$

12. $2(5 - x) + 3(4x - 1) = -6$

13. $4x - 3 = 7x + 6$

14. $-2x + 5 = 3x + 1$

15. $5(2x + 3) - 7 = -2(x + 2)$

16. $8 - 3(2x - 5) = 4(x + 2)$

17. $0.4(2x + 3) = 0.3x + 0.8$

18. $\frac{1}{2}(6x - 5) = x - \frac{3}{2}$

19. **Financial Literacy** Kiera recently bought a used car from a relative, who agreed to let her pay for the car over time. She also borrowed money from her parents for a summer internship in Washington, D.C. She is paying off both loans in equal weekly payments. Use the information in the photos to determine how many weeks it takes for the balances of the loans to be the same.



20. **MP Use Structure** Jan noticed that the equation $3(4x - 7) + 2(4x - 7) = 5$ has the form $3\square + 2\square = 5$ where $\square = 4x - 7$. Explain how Jan can use this observation to solve the equation. Then use the same method to solve the equation $2(3x + 4) - 5(3x + 4) = 33$.
21. **Open Ended** Write an equation that can be solved using the Subtraction Property of Equality and the Division Property of Equality. Show another way to solve the same equation using the Addition Property of Equality and the Multiplication Property of Equality. Explain your reasoning.
22. Two subscription services offer deliveries of boxes of nutritious, organic snack foods each month. For how many months of deliveries will the two plans cost the same?

SNACKS
4 U

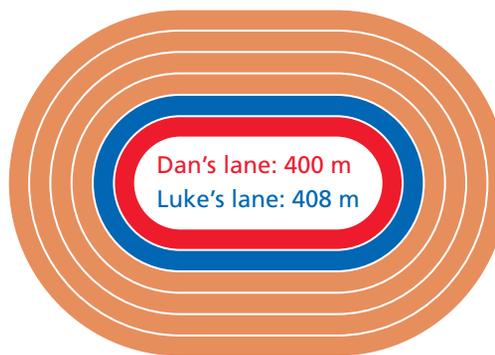
Pay just \$4 per month for the first 3 months of deliveries.

Then pay \$14 per month for deliveries after the third month.

Get 1 free month of deliveries.
Then pay \$10 per month for deliveries after the first month.

HEALTHY
TIMES

23. Dan and Luke begin running on a track at the same time and from the same starting point. Dan's average speed is 4.5 m/s, while Luke's average speed is 5.1 m/s. Dan's running lane is shorter than Luke's lane, as shown. Because Luke runs faster, he will eventually meet up with Dan but be 1 lap ahead of him. How far will Luke have run when this happens?



24. **(MP) Reason** Consider the equation $3(2x - 5) = ax + b$ where a and b represent constants.
- A. For what values of a and b would any value of x be a solution of the equation? Explain your reasoning.
- B. For what values of a and b would the equation not have any solutions? Explain your reasoning.
- C. For what values of a and b would the equation have exactly one solution?
25. **Open Middle™** Using the integers from -9 to 9 at most one time each, replace the boxes to create an equation that has a positive solution. Then repeat this activity to create a second equation that has a negative solution.

$$\frac{\boxed{?}}{\boxed{?}} \left(\boxed{?}x + \boxed{?} \right) + \boxed{?}x = \boxed{?}x + \boxed{?}$$

Spiral Review • Assessment Readiness

26. Which expressions are equivalent to $4x^2y^6$? Select all that apply.
- (A) $(2xy^3)^2$ (D) $2x^2y^6 + 2x^2y^6$
 (B) $(2x^2y^6)^2$ (E) $(16x^8y^{24})^{\frac{1}{4}}$
 (C) $2xy^2 + 2xy^4$ (F) $(16x^4y^{12})^{\frac{1}{2}}$
27. A circular mulch bed has a radius of 1.6 feet. A bag of mulch contains 2 cubic feet of mulch. If all of the mulch is spread evenly on the bed, what is the mulch's depth to an appropriate number of significant digits?
- (A) 0.2 foot (C) 0.249 foot
 (B) 0.25 foot (D) 0.2487 foot
28. To make salsa, you buy 8 tomatoes that weigh t pounds and that cost \$2.20 per pound. You also buy 2 peppers that weigh p pounds and that cost \$3.40 per pound. Which expression gives the total cost (in dollars) of the items?
- (A) $2.2t$ (C) $2.2t + 3.4p$
 (B) $3.4p$ (D) $17.6t + 6.8p$
29. Which formulas can be used to find the perimeter P of a rectangle with length ℓ and width w ? Select all that apply.
- (A) $P = \ell + w$ (D) $P = (2\ell)(2w)$
 (B) $P = \ell w$ (E) $P = 2(\ell + w)$
 (C) $P = 2\ell + 2w$ (F) $P = 4\ell w$



I'm in a **Learning Mindset!**

What properties did I learn to use to justify the steps in solving an equation?

2.2

Write and Solve Equations

LESSON FOCUS AND COHERENCE

Mathematics Standards

- Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.
- Model with mathematics.

I Can Objective

I can use the properties of equality and the Distributive Property to solve an equation with the variable on both sides.

Learning Objective

Solve linear equations with grouping symbols or with the variable on both sides, and use linear equations to model and solve real-world problems.

Language Objective

Explain the steps needed to solve a linear equation with grouping symbols or with the variable on both sides.

Vocabulary

Review: equation, equivalent equations, solution of an equation in one variable

Lesson Materials: spreadsheet software

Mathematical Progressions

Prior Learning	Current Development	Future Connections
<p>Students:</p> <ul style="list-style-type: none"> • wrote and interpreted algebraic expressions that represent real-world quantities. (2.1) • wrote and solved equations of the form $px + q = r$ and $p(x + q) = r$ where p, q, and r are specific rational numbers. (Gr7, 7.3 and 7.4) 	<p>Students:</p> <ul style="list-style-type: none"> • solve linear equations with grouping symbols or with the variable on both sides. • explain and justify each step required to solve a linear equation. • use linear equations to represent and solve real-world problems. 	<p>Students:</p> <ul style="list-style-type: none"> • will solve linear equations with coefficients represented by letters. (2.3) • will write and solve linear inequalities. (2.4 and 2.5)

PROFESSIONAL LEARNING

Visualizing the Math

Students may find it helpful to draw a bar model to represent real-world situations, such as the problem in Task 5 on page 41. The bar model at the right shows that the sum of Jon's distance and Josh's distance equals the length of the trail.

So an equation that models the problem is $15t + 12\left(t + \frac{1}{3}\right) = 22$.

Jon's distance (mi)	Josh's distance (mi)
$15t$	$12\left(t + \frac{1}{3}\right)$
22	
Length of trail (mi)	



ACTIVATE PRIOR KNOWLEDGE • Write and Solve Equations

Use these activities to quickly assess and activate prior knowledge as needed.

Problem of the Day

Mia has a \$25 gift card for an online music store. She buys an album for \$9.50 and wants to use the remaining money on the card to buy individual songs, which cost \$1.25 each. Write and solve an equation to find the number of songs Mia can buy. **Let x represent the number of songs; $9.50 + 1.25x = 25$; $x = 12.4$; Mia can buy 12 songs.**

Quick Check for Homework

As part of your daily routine, you may want to display the Teacher Solution Key to have students check their homework.

Make Connections

Based on students' responses to the Problem of the Day, choose one of the following:

1 Project the Interactive Reteach, Grade 7, Lesson 7.4.

2 Complete the Prerequisite Skills Activity:

Have students work in pairs. One student should write an expression of the form $px + q$, such as $-2x + 3$. The other student should write a constant r , such as 15. Have the two students work together to solve the equation $px + q = r$ and justify each step of their solution method. Then have students switch roles to create and solve a different equation.

- Think about the equation $-2x + 3 = 15$. What is the first step for solving this equation? What property do you use for this step? **Subtract 3 from each side; Subtraction Property of Equality**
- What equation do you get when you subtract 3 from each side of $-2x + 3 = 15$? **$-2x = 12$**
- What is the next step for solving the equation? What property do you use? **Divide each side by -2 ; Division Property of Equality**
- What is the solution of the equation? How can you check the solution? **-6 ; Substitute -6 for x in the original equation $-2x + 3 = 15$ and verify that you get a true statement.**

If students continue to struggle, use Tier 2 Skill 3.

SHARPEN SKILLS

If time permits, use this on-level activity to build fluency and practice basic skills.

Vocabulary Review

Objective: Students demonstrate an understanding of equivalent equations.

Materials: index cards

Have students work in small groups. On half of the index cards, write a two-step equation. On the other cards, write a one step-equation, such that some of the one-step equations are equivalent to one or more of the two-step equations. Give each group a set of each type of card. Ask students to find any equivalent equation pairs in the index cards they are given.

Encourage students to explain in their own words what it means for equations to be equivalent.

$$5x + 8 = 13$$

$$-3g + 2 = 8$$

$$5x = 5$$

$$-g = 2$$

$$-3g = 6$$

$$13x = 13$$

Small-Group Options

Use these teacher-guided activities with pulled small groups.

On Track

Materials: index cards

Give each student a card with a linear expression such as $2x + 4$ or $5(3x + 2)$ written on it. Have students pair up and solve the equation formed by setting their expressions equal to each other. Each student should pair up with as many other students as time allows.

$$2x + 4$$

$$5(3x + 2)$$

Almost There

Materials: algebra tiles

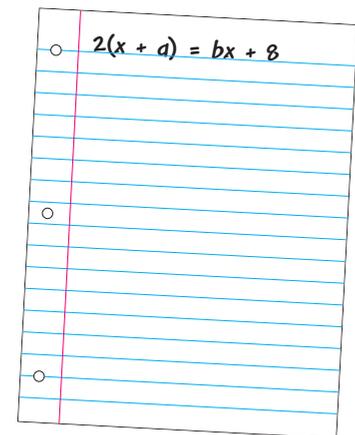
Write the equation $3(x + 2) + 1 = 13$. Have students do the following:

- Model the equation with algebra tiles, using three groups of one x -tile and two 1-tiles to represent $3(x + 2)$.
- Use the model to write an equivalent equation without grouping symbols.
- Solve the equation by performing the same operation on each “side” of the model, such as adding tiles to or removing tiles from each side. Continue until you have an x -tile by itself on one side and only 1-tiles on the other side. After each step, write the corresponding equation.
- Review your solution steps. Then solve the equation $4(x - 5) - 7 = -9$ without using algebra tiles.

Ready for More

Write the equation $2(x + a) = bx + 8$. Have students find values of a and b for which the following are true:

- The equation has infinitely many solutions.
- The equation has no solution.
- The equation has -3 as its only solution.



Math Center Options

Use these student self-directed activities at centers or stations.

Key: ● Print Resources ● Online Resources

On Track

- Interactive Digital Lesson
- Journal and Practice Workbook
- Interactive Glossary: **equation**, **equivalent equations**, **solution of an equation in one variable**
- Module Performance Task: Spies and Analysts

Almost There

- Reteach 2.2
- Interactive Reteach 2.2
- RtI Tier 2 Skill 3: Solve Two-Step Equations

Ready for More

- Challenge 2.2
- Interactive Challenge 2.2

Unit Project Check students' progress by asking what equation they wrote to find the amount of water needed to dilute the solution.



View data-driven recommendations and assign differentiation resources.

During the *Spark Your Learning*, listen and watch for strategies students use. See samples of student work on this page.

Use an Equation

Strategy 1

Let t = Lynn's jogging time in hours.

Let $t - 0.1$ = Anna's jogging time in hours.

$$5.4t = 6(t - 0.1)$$

$$5.4t = 6t - 0.6$$

$$-0.6t = -0.6$$

$$t = 1$$

Anna catches up with Lynn 1 hour after Lynn starts jogging, or at 5:00 p.m.

If students . . . use an equation to solve the problem, they are employing an efficient method and demonstrating an exemplary understanding of writing and solving linear equations from Grade 8.

Have these students . . . explain how they determined their equation and how they solved it. **Ask:**

- Q How did you use the given information and the relationship between speed, time, and distance to write your equation?
- Q What properties did you use to solve your equation?

Use a Table

Strategy 2

Let t = time after 4:00 p.m. in hours.

t	Lynn's Distance (mi)	Anna's Distance (mi)
0	0	0
0.1	$5.4(0.1) = 0.54$	0
0.2	$5.4(0.2) = 1.08$	$6(0.2 - 0.1) = 0.6$
0.5	$5.4(0.5) = 2.7$	$6(0.5 - 0.1) = 2.4$
0.8	$5.4(0.8) = 4.32$	$6(0.8 - 0.1) = 4.2$
0.9	$5.4(0.9) = 4.86$	$6(0.9 - 0.1) = 4.8$
1	$5.4(1) = 5.4$	$6(1 - 0.1) = 5.4$

Anna catches up with Lynn 1 hour after 4:00 p.m., or at 5:00 p.m.

If students . . . use a table to solve the problem, they understand how the quantities in the problem are related but may not know how to model the problem algebraically.

Activate Prior Knowledge . . . by having students write algebraic expressions for the distances jogged by Lynn and Anna. **Ask:**

- Q How can you write expressions with a variable for the distances that Lynn and Anna jog?
- Q What is true about your expressions when Anna catches up with Lynn?

COMMON ERROR: Uses Wrong Units

Let t = Lynn's jogging time in hours.

Let $t - 6$ = Anna's jogging time in hours.

$$5.4t = 6(t - 6)$$

$$5.4t = 6t - 36$$

$$-0.6t = -36$$

$$t = 60$$

Anna catches up with Lynn 60 hours after Lynn starts jogging.

If students . . . do not convert 6 minutes to 0.1 hour when writing an expression for Anna's jogging time, they may not understand that the same units for time must be used for all quantities in the equation.

Then intervene . . . by pointing out that since the problem gives time in different units, students must convert times to a common unit. **Ask:**

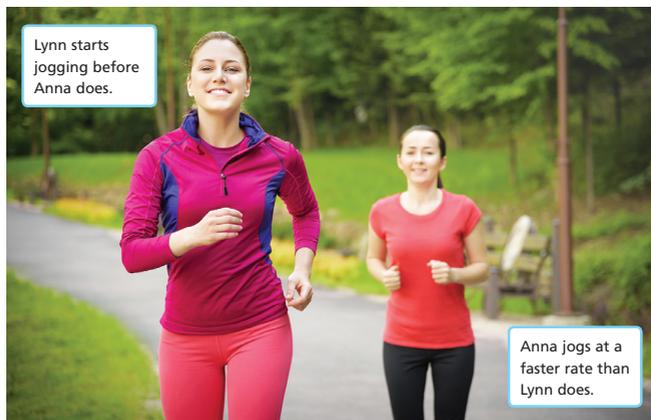
- Q The speeds in the problem are given in miles per hour. If you use these speeds in your equation, in what units must you express the jogging times?
- Q If you calculate that Anna takes 60 hours to catch up with Lynn, why might you think you have made a mistake?

Write and Solve Equations

I Can use the properties of equality and the Distributive Property to solve an equation with the variable on both sides.

Spark Your Learning

Lynn and Anna are joggers who use the same running trail.



Complete Part A as an entire class. Then complete Parts B–D in small groups.

- A.** What mathematical question can you ask about this situation? What information would you need to know to answer your question? **A. When will Anna catch up with Lynn?; how fast each woman jogs and when each woman started jogging**
- B.** What variable(s) are involved in this situation? What unit of measurement would you use for each variable? **Possible answer: time; hours**
- C.** How would you use the information in the photo and the additional information your teacher gave you to answer your question? What is the answer? **See Strategy 1 on the facing page.**
- D.** Does your answer make sense in the context of the situation? How do you know? **yes; Possible answer: One hour is a reasonable amount of time for a person to jog.**



Turn and Talk Predict how your answer would change for each of the following changes in the situation: **See margin.**

- Anna starts jogging 3 minutes earlier.
- Lynn's average jogging speed is 0.2 mile per hour faster.
- Lynn and Anna jog at the same average speed.

Image Credit: ©iStock.com/Shutterstock

Module 2 • Lesson 2.2

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1 Spark Your Learning

MOTIVATE

- Have students look at the photo in their books and read the information contained in the photo. Then complete Part A as a whole-class discussion.
- Give the class the additional information they need to solve the problem. This information is available online as a printable and projectable page in the Teacher Resources.
- Have students work in small groups to complete Parts B–D.

PERSEVERE

If students need support, guide them by asking:

- Q Assessing** When Anna catches up with Lynn, what is true of the distances both people have jogged? **The distances are equal.**
- Q Assessing** How can you write an expression for the distance each person jogs? **I can write the product of the person's speed and her jogging time. I will need to use a variable or a variable expression for the jogging time since this information is unknown.**
- Q Advancing** How can you use your expressions for the distances Lynn and Anna jog to solve the problem? **I can set the expressions equal to each other to form an equation. I can then solve the equation to find the time when Anna catches up with Lynn.**



Turn and Talk When predicting how their answers would change, students should use reasoning to determine the direction of the change rather than compute the new answers. **Possible answer: If Anna starts jogging 3 minutes earlier, she will catch up with Lynn sooner. If Lynn's average jogging speed is 0.2 mi/h faster, Anna will take longer to catch up with her. If Lynn and Anna jog at the same average speed, Anna will never catch up with Lynn.**

BUILD SHARED UNDERSTANDING

Select groups of students who used various strategies and have them share with the class how they solved the problem. Encourage students to ask questions of each other. Make sure students understand that since the joggers' speeds are given in miles per hour, the jogging times must be expressed in hours. This means that the difference between Lynn's and Anna's starting times, 6 minutes, must be converted to 0.1 hour.

CULTIVATE CONVERSATION • Co-Craft Questions

If students have difficulty formulating a mathematical question about the situation in the Spark Your Learning, ask them to visualize themselves running after a friend who is ahead of them but moving at a slower speed. What are some natural questions to ask about this situation?

Work together to craft the following questions:

- Will Anna catch up with Lynn?
- How much time will it take Anna to catch up with Lynn?
- What distance will Anna and Lynn have traveled when Anna catches up with Lynn?

Then have students think about what additional information, if any, they would need to answer these questions. **Ask:**

- Can you determine when Anna catches up with Lynn if you know only the joggers' speeds? Why or why not?
- Can you determine when Anna catches up with Lynn if you know only the time at which each person started jogging? Explain.

2 Learn Together

Build Understanding

Task 1  **Use Tools** Students use if-then-else statements in a spreadsheet as a means of building equivalent equations using properties of equality.

CONNECT TO VOCABULARY

Have students use the **interactive glossary** to record their understanding of the vocabulary in this task.

Sample Guided Discussion:

Q In Part B, the spreadsheet shows that $x = -1$ and $2x = -2$ have the same solution, -1 . Why would you expect this? The equation $2x = -2$ was obtained by multiplying each side of $x = -1$ by 2. The Multiplication Property of Equality says that both equations are equivalent, so they have the same solution.



Turn and Talk Help students understand that to solve $2x + 3 = 1$, they need to reverse the steps they used in Task 1. They should realize that subtracting “undoes” addition and that division “undoes” multiplication. Starting with $2x + 3 = 1$, I can first use the Subtraction Property of Equality to subtract 3 from each side, giving me $2x = -2$. Then I can use the Division Property of Equality to divide each side by 2, giving me $x = -1$. So the solution of $2x + 3 = 1$ is -1 .

Build Understanding

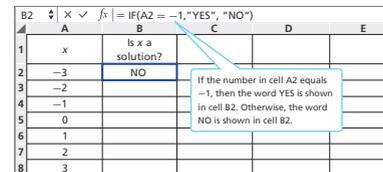
Investigate Properties of Equality

An **equation** is a mathematical statement comparing two expressions using the symbol $=$. When you solve an equation, the **solution** produces a true statement when substituted for the variable in the equation. Solving an equation involves using properties of equality to write simpler **equivalent equations**, which all have the same solution as the original equation. In the Symbols column of the table below, a , b , and c are real numbers.

Properties of Equality		
Property	Words	Symbols
Addition Property of Equality	Adding the same number to both sides of an equation produces an equivalent equation.	If $a = b$, then $a + c = b + c$. Example: If $x - 2 = 3$, then $x - 2 + 2 = 3 + 2$.
Subtraction Property of Equality	Subtracting the same number from both sides of an equation produces an equivalent equation.	If $a = b$, then $a - c = b - c$. Example: If $x + 4 = -1$, then $x + 4 - 4 = -1 - 4$.
Multiplication Property of Equality	Multiplying both sides of an equation by the same nonzero number produces an equivalent equation.	If $a = b$ and $c \neq 0$, then $ac = bc$. Example: If $\frac{x}{3} = 2$, then $\frac{x}{3} \cdot 3 = 2 \cdot 3$.
Division Property of Equality	Dividing both sides of an equation by the same nonzero number produces an equivalent equation.	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. Example: If $-2.5x = 10$, then $\frac{-2.5x}{-2.5} = \frac{10}{-2.5}$.

1 A-C. See Additional Answers.

A. Using a spreadsheet, create a table like the one shown for the equation $x = -1$. The formula in cell B2 is an if-then-else statement that checks the value in cell A2. Fill down the formula from cell B2 to cell B8. What do you observe?



	A	B	C	D	E
1	x	=IF(A2=-1,"YES","NO")			
2	-3	NO			
3	-2	YES			
4	-1	YES			
5	0	NO			
6	1	NO			
7	2	NO			
8	3	NO			

- B.** Suppose you use the Multiplication Property of Equality to rewrite the equation $x = -1$ as $2x = -2$. Use the new equation in if-then-else statements you enter in cells C2 through C8. What do you observe?
- C.** Suppose you use the Addition Property of Equality to rewrite the equation $2x = -2$ as $2x + 3 = 1$. Use the new equation in if-then-else statements you enter in cells D2 through D8. What do you observe?



Turn and Talk You used properties of equality to build the equation $2x + 3 = 1$ from $x = -1$. How can you use properties of equality to solve the equation $2x + 3 = 1$? **See margin.**

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LEVELED QUESTIONS

Depth of Knowledge (DOK)	Leveled Questions	What Does This Tell You?
Level 1 Recall	What property allows you to subtract 1 from each side of $3x + 1 = -5$ to obtain the equivalent equation $3x = -6$? Subtraction Property of Equality	Students' answers will indicate whether they understand the meaning of one of the properties of equality.
Level 2 Basic Application of Skills & Concepts	How can you solve the equation $3x + 1 = -5$? I can use the Subtraction Property of Equality to subtract 1 from each side, giving $3x = -6$. Then I can use the Division Property of Equality to divide each side by 3, giving the solution $x = -2$.	Students' answers will demonstrate whether they can use the properties of equality to solve a two-step equation.
Level 3 Strategic Thinking & Complex Reasoning	How can you show that $3x + 1 = -5$ and $4x + 3 = -9$ are <i>not</i> equivalent equations? Possible answer: I can solve $3x + 1 = -5$ to get $x = -2$ and $4x + 3 = -9$ to get $x = -3$. The solutions are different, so, the equations are not equivalent.	Students' answers will reflect whether they understand the meaning of equivalent equations and can reason strategically about how to determine whether two equations are equivalent.

Step It Out

Solve Equations Using the Distributive Property

Previously, you solved one-step and two-step equations. Now you will solve multistep equations. Such equations may contain grouping symbols. In order to free the terms in an equation that contains grouping symbols, you can use the Distributive Property.

- 2** The steps for solving the equation $5(2x - 3) + 4 = -6$ and checking the solution are shown below, but the steps have been scrambled.

A. Write the solution steps in the correct order.

$10x - 11 = -6$
$x = 0.5$
$10x - 15 + 4 = -6$
$5(2x - 3) + 4 = -6$
$10x = 5$

B. Write the check steps in the correct order.

$-6 = -6$ ✓
$5(2(0.5) - 3) + 4 \stackrel{?}{=} -6$
$5(-2) + 4 \stackrel{?}{=} -6$
$5(1 - 3) + 4 \stackrel{?}{=} -6$
$-10 + 4 \stackrel{?}{=} -6$

A, B. See Additional Answers.



Turn and Talk Can you solve the equation $5(2x - 3) + 4 = -6$ without using the Distributive Property as one of the steps? Show how or explain why not. **See margin.**

Recall that the Distributive Property also allows you to combine like terms.

- 3** The steps for solving the equation $-2x + \frac{1}{2}(6x - 5) = \frac{3}{2}$ are shown, but some justifications are missing.

$$\begin{aligned} -2x + \frac{1}{2}(6x - 5) &= \frac{3}{2} \\ -2x + 3x - \frac{5}{2} &= \frac{3}{2} \\ (-2 + 3)x - \frac{5}{2} &= \frac{3}{2} \\ x - \frac{5}{2} &= \frac{3}{2} \\ x &= 4 \end{aligned}$$

Given equation

?

?

Simplify the coefficient of x .

?

A. Distributive Property

A. What property justifies rewriting $\frac{1}{2}(6x - 5)$ as $3x - \frac{5}{2}$?

B. What property justifies rewriting $-2x + 3x$ as $(-2 + 3)x$?

B. Distributive Property

C. What property justifies adding $\frac{5}{2}$ to each side of the equation?

C. Addition Property of Equality



Turn and Talk Is it possible to eliminate the fractions as a first step in solving the equation $-2x + \frac{1}{2}(6x - 5) = \frac{3}{2}$? Show how or explain why not. **See margin.**

EL

PROFICIENCY LEVEL

Beginning

Write the terms $-2x$ and $3x$. Say, "These are like terms because each term has the same variable, x , raised to the same power—the first power." Then write the term $4m^2$ and ask students to write a like term for $4m^2$.

Intermediate

Have students work in groups. Give each group a set of index cards. Each card should show two terms, such as $5x$ and $-3x$ or $6y^3$ and $6y^2$. Ask students to explain why the terms on each card either are or are not like terms.

Advanced

Have students explain how to use the Distributive Property to combine like terms.

Step It Out

Task 2



Use Structure Encourage students to use the structure of addition to check whether the Distributive Property works for subtraction.

By writing subtraction as adding the opposite of a number, students can demonstrate that the Distributive Property also holds for a product of a number and a difference of two numbers:

$$a(b - c) = a[b + (-c)] = ab + a(-c) = ab - ac$$

Sample Guided Discussion:

- Q** What will be the last equation in the sequence of solution steps? Why? **The equation $x = 0.5$ will be last because it is the only equation that has the variable by itself on one side.**
- Q** Why do you substitute the solution you found into the original equation to check your work? **Possible answer: I could have made a mistake when solving the original equation. If so, some of my intermediate equations may contain errors.**



Turn and Talk Students may not know how to solve the equation without the Distributive Property. Point out that they can first follow steps to isolate the expression in parentheses, and then work to isolate the variable. **yes; I can subtract 4 from each side to get $5(2x - 3) = -10$. Then I can divide each side by 5 to get $2x - 3 = -2$. Next, I can add 3 to each side to get $2x = 1$. Finally, I can divide each side by 2 to get $x = 0.5$.**

Task 3



Use Structure Encourage students to use the structure of the Distributive Property and the properties of equality to justify each step in the solution.

Sample Guided Discussion:

- Q** On the left side of the second equation, why is $\frac{5}{2}$ instead of 5 being subtracted? **When you use the Distributive Property to rewrite $\frac{1}{2}(6x - 5)$, you must multiply both terms inside the parentheses by $\frac{1}{2}$, not just the first term.**



Turn and Talk Help students realize that they can eliminate the fractions from an equation by multiplying each side by the LCD of the fractions. **yes; I can multiply both sides of the equation by 2 to eliminate the fractions. This gives the equation $-4x + 6x - 5 = 3$.**

Task 4 **MP Use Structure** Students use properties of equality to isolate the variable on one side of an equation.

Point out to students that when solving an equation with the variable on both sides, they can isolate the variable on either the left or right side. Some students may prefer always to isolate the variable on the left side. Others may find it helpful to choose the side that makes the coefficient of the variable be positive when the variable term is isolated on that side.

Sample Guided Discussion:

Q How are the first and second equations in the solution different? How does this suggest the property that was used to obtain the second equation? **The first equation has parentheses while the second equation does not; This suggests that the Distributive Property was used to eliminate the parentheses.**

Q For the equation $4x + 5 = -3x - 9$, what operation do you need to perform so that the variable appears only on the left side? Which property are you using? **Add $3x$ to each side; Addition Property of Equality**



Turn and Talk The equations that students have seen so far in this lesson each have one solution. In this problem, students analyze equations that have no solution or infinitely many solutions. **The original equation, $4(x + 1) + 1 = -3(x + 3)$ has one solution, $x = -2$. When I try to solve $4(x + 1) + 1 = 4(x + 3)$ for x , I get the false equation $5 = 12$, which means this given equation has no solution. When I try to solve $4(x + 1) + 8 = 4(x + 3)$ for x , I get the true equation $12 = 12$, which means that any real number is a solution of this given equation.**

Solve Equations with the Variable on Both Sides

When a variable appears on both sides of an equation, you can use the Addition Property of Equality or the Subtraction Property of Equality to move the variable from one side to the other. Doing so allows you to isolate the variable on one side.

4 The steps for solving the equation $4(x + 1) + 1 = -3(x + 3)$ are shown, but some justifications are missing.

$4(x + 1) + 1 = -3(x + 3)$	Given equation	A. Distributive Property
$4x + 4 + 1 = -3x - 9$?	A. What property lets you rewrite each side without grouping symbols?
$4x + 5 = -3x - 9$	Combine constants.	B. What property justifies adding $3x$ to each side?
$4x + 5 + 3x = -3x - 9 + 3x$?	B. Addition Property of Equality
$7x + 5 = -9$	Combine like terms.	C. What property justifies subtracting 5 from each side?
$7x = -14$?	C. Subtraction Property of Equality
$x = -2$	Division Property of Equality	

Turn and Talk How is solving $4(x + 1) + 1 = 4(x + 3)$ different from solving $4(x + 1) + 1 = -3(x + 3)$? What is different about solving $4(x + 1) + 8 = 4(x + 3)$? **See margin.**

Use an Equation to Solve a Real-World Problem

When solving a real-world problem, you may have to use a formula, such as the formula $P = 2\ell + 2w$ for the perimeter of a rectangle or the formula $d = rt$ for the distance traveled at a constant rate. When using a formula, you should pay attention to the units of measurement associated with the variables to ensure that the units are consistent.

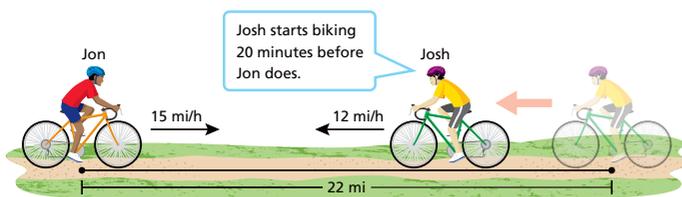
Perimeter ?	=	2 •	Length (meters)	+	2 •	Width ?
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Since the length is in meters, the width must also be in meters so you can add the units on the right side. This means the perimeter is in meters because meters + meters = meters.

Distance (feet)	=	Rate ?	•	Time (seconds)
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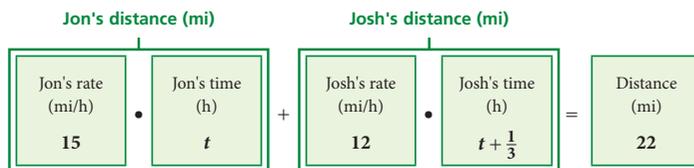
For the units to be consistent in this equation, the rate must be measured in feet per second and not, say, miles per hour.

- 5 Two friends, Jon and Josh, live at opposite ends of a trail. They bike toward each other at the speeds shown. At what distance along the trail, measured from Jon's starting point, do the two friends meet?



Use a verbal model.

Although you are asked to find distance, you will do so by expressing distance in terms of time. Note that the units *miles per hour* and *minutes* are incompatible. So, let t represent the time in *hours* that Jon spends biking.



Write an equation.

$$15t + 12\left(t + \frac{1}{3}\right) = 22$$

Solve the equation.

$$15t + 12\left(t + \frac{1}{3}\right) = 22 \quad \text{Given equation}$$

$$15t + 12t + 4 = 22 \quad \text{Distributive Property}$$

$$27t + 4 = 22 \quad \text{Combine like terms.}$$

$$27t = 18 \quad \text{Subtraction Property of Equality}$$

$$t = \frac{2}{3} \quad \text{Division Property of Equality}$$

Answer the question.

The two friends will meet at a distance of $15\left(\frac{2}{3}\right) = 10$ miles from Jon's starting point on the trail.

A. Why is this expression $t + \frac{1}{3}$ and not $t + 20$?

A. Josh's time is measured in hours, not minutes.

B. How can you use the solution of the equation to find the distance from Jon's starting point? What is this distance?

B. See Additional Answers.



Turn and Talk Suppose the trail in Task 5 is only 3 miles long. Write and solve a new equation to find the time Jon spends biking. Does your solution make sense? Explain. **See margin.**

Task 5 **MP Model with Mathematics** Encourage students to use a verbal model to help them write a mathematical model for a real-world situation.

EL SUPPORT SENSE-MAKING Three Reads

Have students read the problem three times. Use the questions below for a different focus each read.

- 1 What is the situation about?
- 2 What are the quantities in the situation?
- 3 What are the possible mathematical questions that you could ask for the situation?

Sample Guided Discussion:

- Q** What is true about the sum of Jon's biking distance and Josh's biking distance? **The sum of the distances is 22 miles.**
- Q** How can you express the biking distances in terms of time? **You can write each distance as the product of speed and time.**
- Q** How can you convert 20 minutes to hours? **Possible answer: I can multiply 20 minutes by the conversion factor $\frac{1 \text{ hour}}{60 \text{ minutes}}$ to obtain the equivalent time $\frac{1}{3}$ hour.**
- Q** The solution of the equation in the problem is $t = \frac{2}{3}$ hour. Is this the answer to the question asked in the problem? Explain. **no; The problem asks for the distance from Jon's starting point at which Jon and Josh meet. You need to multiply the solution of the equation by Jon's speed, 15 mi/h, to find this distance and answer the question.**



Turn and Talk If students do not notice that the solution of the equation makes no sense, ask them if time can be negative. **The equation would be $15t + 12\left(t + \frac{1}{3}\right) = 3$, so $27t + 4 = 3$, and $t = -\frac{1}{27}$, but time cannot be negative. So, Jon and Josh cannot be 3 miles apart when they start.**



Assign the Digital On Your Own for

- built-in student supports
- Actionable Item Reports
- Standards Analysis Reports

On Your Own

Assignment Guide

The chart below indicates which problems in the On Your Own are associated with each task in the Learn Together. Assign daily homework for tasks completed.

Learn Together Tasks	On Your Own Problems
Task 1, p. 38	Problems 5 and 21
Task 2, p. 39	Problems 7–10
Task 3, p. 39	Problems 11, 12, and 20
Task 4, p. 40	Problems 13–18, 24, and 25
Task 5, p. 41	Problems 6, 19, 22, and 23

ANSWERS

- 6.A.** Let p represent the manager's new pay rate. Then $p - 4$ represents the manager's old pay rate; $35p - 30(p - 4) = \$197.50$; \$15.50 per hour
- 6.B.** Let p represent the manager's old pay rate. Then $p + 4$ represents the manager's new pay rate; $35(p + 4) - 30p = \$197.50$; \$11.50 per hour

Check Understanding

1. Which two of the equations shown below are equivalent? Explain your reasoning. $2x + 7 = 1$, $x = -3$; **Both equations have the same solution, -3 .**

$$2x + 7 = 1 \qquad -3x + 4 = 12 \qquad x = -3$$

- Solve each equation. Justify your solution steps, and check each solution. **2, 3. See Additional**

2. $6(2x - 5) - 8 = 4$ $x = \frac{7}{2}$ 3. $\frac{1}{2}(4x - 3) = \frac{3}{4}(4x - 5)$ $x = \frac{9}{4}$

Answers for solution steps, justifications, and solution checks.

4. Membership at the local art center is \$60 per year. If you are a member, you can take lessons at the center for \$7 each. If you are not a member, lessons cost \$10 each. For how many lessons is the total cost with a membership the same as the total cost of lessons without a membership? **20 lessons**

On Your Own

5. **MP Critique Reasoning** Mark solved the equation $\frac{2}{3}x = 9$ by using the Division Property of Equality to get $x = 6$. Did Mark solve the equation correctly? Explain why or why not. **no; Mark multiplied 9 by $\frac{2}{3}$ instead of dividing 9 by $\frac{2}{3}$.**

Total pay increase: \$197.50

6. **MP Use Repeated Reasoning** Melissa used the information shown at the right and the guess and check method to find the manager's new pay rate after being promoted.

A. Melissa's work is shown in the table below. Explain how she can generalize what she has done by defining a variable and then writing and solving an equation using that variable.



- 6. A, B. See margin.**

New pay rate guess	Calculated new weekly pay	Calculated old weekly pay	Calculated increase in weekly pay
\$12.00 per hour	$35(\$12.00) = \420.00	$30(\$12.00 - \$4.00) = \$240.00$	$\$420.00 - \$240.00 = \$180.00$ ✗
\$15.00 per hour	$35(\$15.00) = \525.00	$30(\$15.00 - \$4.00) = \$330.00$	$\$525.00 - \$330.00 = \$195.00$ ✗
\$16.00 per hour	$35(\$16.00) = \560.00	$30(\$16.00 - \$4.00) = \$360.00$	$\$560.00 - \$360.00 = \$200.00$ ✗
\$15.50 per hour	$35(\$15.50) = \542.50	$30(\$15.50 - \$4.00) = \$345.00$	$\$542.50 - \$345.00 = \$197.50$ ✓

- B. Suppose the question had been, "What was the manager's old pay rate?" Define the variable in a way that will answer this question. Then write and solve a new equation to show that you get a solution that is consistent with the one from Part A.

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3 Check Understanding

Formative Assessment

Use formative assessment to determine if your students are successful with this lesson's learning objective.

Students who successfully complete the Check Understanding can continue to the On Your Own practice.

For students who miss 1 problem or more, work in a pulled small group using the Almost There small-group activity on page 37C.



Assign the Digital Check Understanding to determine

- success with the learning objective
- items to review
- grouping and differentiation resources

4 Differentiation Options

Differentiate instruction for all students using small-group activities and math center activities on page 37C.

Reteach

Write and Solve Equations

Complete the solution by providing the missing information.

Use algebra:

Use tables:

Use graphs:

Challenge

Solve Simple Quadratic and Cubic Equations

1. Solve the equation $2x^2 + 11x + 12 = 0$. Justify your steps.

2. Solve the equation $3x^2 + 11x + 6 = 0$. Justify your steps.

3. The equation $3x^2 + 11x + 6 = 0$ is a quadratic equation. Solve the equation for x . Justify your steps.

4. Recall the definition of opposite reciprocals. A line with slope m and y -intercept b is perpendicular to a line with slope $-\frac{1}{m}$ and y -intercept c . Write the equation of the line that is perpendicular to the line $2x + 3y = 6$ and passes through the point $(1, 1)$.

5. What happens when you solve the equation $2x^2 + 11x + 6 = 0$ for x ?

6. Recall the definition of cube roots. A cube root of a real number a is a real number b such that $b^3 = a$. Write the equation of the line that is perpendicular to the line $2x + 3y = 6$ and passes through the point $(1, 1)$.

7. What happens when you solve the equation $2x^2 + 11x + 6 = 0$ for x ?

7–18. See Additional Answers for solution steps, justifications, and solution checks.

Solve each equation. Justify your solution steps, and check each solution.

7. $4(3x - 10) + 7 = 15$ $x = 4$
 8. $6 - 5(2x + 1) = 21$ $x = -2$
 9. $2 - \frac{3}{4}(8x - 6) = 11$ $x = -\frac{3}{4}$
 10. $0.2(4 - 5x) + 1 = 2.4$ $x = -0.6$
 11. $5(x + 1) - 2(3x - 4) = 14$ $x = -1$
 12. $2(5 - x) + 3(4x - 1) = -6$ $x = -1.3$
 13. $4x - 3 = 7x + 6$ $x = -3$
 14. $-2x + 5 = 3x + 1$ $x = 0.8$
 15. $5(2x + 3) - 7 = -2(x + 2)$ $x = -1$
 16. $8 - 3(2x - 5) = 4(x + 2)$ $x = 1.5$
 17. $0.4(2x + 3) = 0.3x + 0.8$ $x = -0.8$
 18. $\frac{1}{2}(6x - 5) = x - \frac{3}{2}$ $x = \frac{1}{2}$

19. **Financial Literacy** Kiera recently bought a used car from a relative, who agreed to let her pay for the car over time. She also borrowed money from her parents for a summer internship in Washington, D.C. She is paying off both loans in equal weekly payments. Use the information in the photos to determine how many weeks it takes for the balances of the loans to be the same. **28 weeks**



20. **MP Use Structure** Jan noticed that the equation $3(4x - 7) + 2(4x - 7) = 5$ has the form $3\square + 2\square = 5$ where $\square = 4x - 7$. Explain how Jan can use this observation to solve the equation. Then use the same method to solve the equation $2(3x + 4) - 5(3x + 4) = 33$. **See margin.**

21. **Open Ended** Write an equation that can be solved using the Subtraction Property of Equality and the Division Property of Equality. Show another way to solve the same equation using the Addition Property of Equality and the Multiplication Property of Equality. Explain your reasoning. **See margin.**

22. Two subscription services offer deliveries of boxes of nutritious, organic snack foods each month. For how many months of deliveries will the two plans cost the same?
5 months

**SNACKS
4 U**

Pay just \$4 per month for the first 3 months of deliveries.
Then pay \$14 per month for deliveries after the third month.

Get 1 free month of deliveries.
Then pay \$10 per month for deliveries after the first month.

**HEALTHY
TIMES**

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Questioning Strategies

Problem 19 Suppose *both* loans are paid off at a rate of \$200 per week. Without doing any calculations, what conclusion can you draw about when the loan balances will be the same? **The loan balances will never be the same. Because the \$6500 loan has a larger initial balance but is paid off at the same rate as the smaller loan, the larger loan's balance will never "catch up" with the smaller loan's balance.**

Watch for Common Errors

Problem 22 Some students may write the expressions for the subscription costs as $4 + 14m$ for Snacks 4 U and $10m$ for Healthy Times, where m is the subscription time in months. Point out that the cost of \$14 per month for Snacks 4 U applies only to the amount of time over 3 months, so the correct expression for Snacks 4 U is $4 + 14(m - 3)$. Similarly, the cost of \$10 per month for Healthy Times applies only to the amount of time after 1 month, so the correct expression for Healthy Times is $10(m - 1)$. Therefore, the equation for the problem is $4 + 14(m - 3) = 10(m - 1)$.

ANSWERS

20. For $3(4x - 7) + 2(4x - 7) = 5$: $3\square + 2\square = 5$ implies that $5\square = 5$, or $\square = 1$. Since $\square = 4x - 7$, this means that $4x - 7 = 1$, or $4x = 8$, or $x = 2$.

For $2(3x + 4) - 5(3x + 4) = 33$: $2\square - 5\square = 33$ implies that $-3\square = 33$, or $\square = -11$. Since $\square = 3x + 4$, this means $3x + 4 = -11$, or $3x = -15$, or $x = -5$.

21. Possible answer:
Using Subtraction and Division: $3x + 4 = 13$;
 $3x + 4 - 4 = 13 - 4$; $3x = 9$; $\frac{3x}{3} = \frac{9}{3}$; $x = 3$
Using Addition and Multiplication: $3x + 4 = 13$;
 $3x + 4 + (-4) = 13 + (-4)$; $3x = 9$; $\frac{1}{3}(3x) = \frac{1}{3}(9)$; $x = 3$

5 Wrap-Up

Summarize learning with your class. Consider using the Exit Ticket, Put It in Writing, or I Can scale.

Exit Ticket

Kyla alternates between walking and jogging on an 11-mile trail. Her walking speed is 4 mi/h, and her jogging speed is 6 mi/h. Kyla takes 2 hours to go from the beginning to the end of the trail. Write and solve an equation to find the distance Kyla walked and the distance she jogged.

$4t + 6(2 - t) = 11$; $t = 0.5$; Kyla walked $4(0.5) = 2$ miles and jogged $6(2 - 0.5) = 9$ miles.

Put It in Writing

Describe some strategies you can use to solve a linear equation in one variable.

I Can

The scale below can help you and your students understand their progress on a learning goal.

4

I can use the properties of equality and the Distributive Property to solve an equation with the variable on both sides, and I can explain my solution steps to others.

3

I can use the properties of equality and the Distributive Property to solve an equation with the variable on both sides.

2

I can use the properties of equality and the Distributive Property to solve an equation with the variable and grouping symbols on one side.

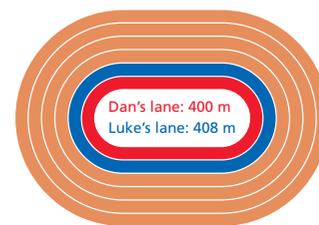
1

I can recognize which property of equality is being used to produce an equivalent equation in a stepped-out solution.

Spiral Review • Assessment Readiness

These questions will help determine if students have retained information taught in the past and can also prepare them for high-stakes assessments. Here, students must recognize equivalent expressions or formulas (2.1), choose a level of precision appropriate to limitations on measurement when reporting a quantity (1.3), and interpret the meaning of algebraic expressions in the context of a problem situation (2.1).

23. Dan and Luke begin running on a track at the same time and from the same starting point. Dan's average speed is 4.5 m/s, while Luke's average speed is 5.1 m/s. Dan's running lane is shorter than Luke's lane, as shown. Because Luke runs faster, he will eventually meet up with Dan but be 1 lap ahead of him. How far will Luke have run when this happens? **4080 meters**



24. **(MP) Reason** Consider the equation $3(2x - 5) = ax + b$ where a and b represent constants.
- For what values of a and b would any value of x be a solution of the equation? Explain your reasoning. **A–C. See Additional Answers.**
 - For what values of a and b would the equation not have any solutions? Explain your reasoning.
 - For what values of a and b would the equation have exactly one solution?
25. **Open Middle™** Using the integers from -9 to 9 at most one time each, replace the boxes to create an equation that has a positive solution. Then repeat this activity to create a second equation that has a negative solution. **See Additional Answers.**

$$\begin{array}{|c|} \hline ? \\ \hline \end{array} \left(\begin{array}{|c|} \hline ? \\ \hline \end{array} x + \begin{array}{|c|} \hline ? \\ \hline \end{array} \right) + \begin{array}{|c|} \hline ? \\ \hline \end{array} x = \begin{array}{|c|} \hline ? \\ \hline \end{array} x + \begin{array}{|c|} \hline ? \\ \hline \end{array}$$

Spiral Review • Assessment Readiness

26. Which expressions are equivalent to $4x^2y^6$? Select all that apply.
- | | |
|--------------------------------------------|----------------------------------------------------------|
| <input type="checkbox"/> A $(2xy^3)^2$ | <input type="checkbox"/> D $2x^2y^6 + 2x^2y^6$ |
| <input type="checkbox"/> B $(2x^2y^6)^2$ | <input type="checkbox"/> E $(16x^8y^{24})^{\frac{1}{4}}$ |
| <input type="checkbox"/> C $2xy^2 + 2xy^4$ | <input type="checkbox"/> F $(16x^4y^{12})^{\frac{1}{2}}$ |
27. A circular mulch bed has a radius of 1.6 feet. A bag of mulch contains 2 cubic feet of mulch. If all of the mulch is spread evenly on the bed, what is the mulch's depth to an appropriate number of significant digits?
- | | |
|--------------------------------------|----------------------------------------|
| <input type="checkbox"/> A 0.2 foot | <input type="checkbox"/> C 0.249 foot |
| <input type="checkbox"/> B 0.25 foot | <input type="checkbox"/> D 0.2487 foot |
28. To make salsa, you buy 8 tomatoes that weigh t pounds and that cost \$2.20 per pound. You also buy 2 peppers that weigh p pounds and that cost \$3.40 per pound. Which expression gives the total cost (in dollars) of the items?
- | | |
|-----------------------------------|-------------------------------------------|
| <input type="checkbox"/> A $2.2t$ | <input type="checkbox"/> C $2.2t + 3.4p$ |
| <input type="checkbox"/> B $3.4p$ | <input type="checkbox"/> D $17.6t + 6.8p$ |
29. Which formulas can be used to find the perimeter P of a rectangle with length ℓ and width w ? Select all that apply.
- | | |
|---------------------------------------------|----------------------------------------------|
| <input type="checkbox"/> A $P = \ell + w$ | <input type="checkbox"/> D $P = (2\ell)(2w)$ |
| <input type="checkbox"/> B $P = \ell w$ | <input type="checkbox"/> E $P = 2(\ell + w)$ |
| <input type="checkbox"/> C $P = 2\ell + 2w$ | <input type="checkbox"/> F $P = 4\ell w$ |



I'm in a Learning Mindset!

What properties did I learn to use to justify the steps in solving an equation?

44

Keep Going  Journal and Practice Workbook

Learning Mindset



Resilience Monitors Knowledge and Skills

Point out that making sure a given step in solving a problem follows logically from the previous step is essential in all areas of mathematics, not just when solving equations. Encourage students to develop the habit of justifying the steps of their solutions when solving any mathematical problem. Also let students know that although they used the Distributive Property and the properties of equality to solve only linear equations in this lesson, they will use these properties to solve other types of equations later in the course. *How does justifying each step when solving a mathematical problem help you have a more confident mindset? When you can't think of a good justification for one of your solution steps, what might that suggest about your approach to solving the problem?*